

Name : MARK SCHEME	Class	Index Number

METHODIST GIRLS' SCHOOL

Founded in 1887



PRELIMINARY EXAMINATION 2021 Secondary 4

TUESDAY

MATHEMATICS

4048/01

3 August 2021

Paper 1

2 hours

Write your name, class and index number in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.

If working is needed for any question, it must be shown with the answer.

Omission of essential working will result in loss of marks.

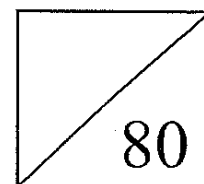
The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give your answer in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 80.



Mathematical Formulae*Compound Interest*

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4 \pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of a triangle} = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

Answer **all** the questions

- 1 Express the ratio $3\frac{2}{3}$ kg : 450 g : 3×10^2 g in its simplest form.

$$\frac{11}{3} : 0.45 : 0.3 \quad \text{M1}$$

$$220 : 27 : 18 \quad \text{A1}$$

Answer ...220... : ...27... : ...18.... [2]

- 2 Given that x is 35% of y , find the value of $\frac{2x}{5y}$, expressing your answer as a fraction in its lowest terms.

$$x = 0.35y$$

$$\frac{x}{y} = 0.35 \quad \text{M1}$$

$$\frac{2x}{5y} = \frac{2}{5} \times 0.35$$

$$= \frac{7}{50} \quad \text{A1}$$

Answer ... $\frac{7}{50}$ [2]

- 3 If $10^m = 2$ and $10^n = 3$, find the value of 10^{3m-n} .

$$10^{3m-n} = \frac{10^{3m}}{10^n} \quad \text{M1}$$

$$= \frac{2^3}{3}$$

$$= \frac{8}{3} \text{ or } 2\frac{2}{3}, \quad \text{A1}$$

Answer $\frac{8}{3}$ or $2\frac{2}{3}$ [2]

4 Simplify $5a^2 - (3a - 2b)^2$.

$$\begin{aligned} & 5a^2 - (3a - 2b)^2 \\ &= 5a^2 - (9a^2 - 12ab + 4b^2) \text{ M1} \\ &= 5a^2 - 9a^2 + 12ab - 4b^2 \\ &= -4a^2 + 12ab - 4b^2 \quad \text{A1} \end{aligned}$$

Answer $-4a^2 + 12ab - 4b^2$ [2]

5 Write as a single fraction in its simplest form $\frac{-4}{x^2-4} + \frac{1}{x-2}$.

$$\begin{aligned} & \frac{-4}{x^2-4} + \frac{1}{x-2} \\ &= \frac{-4}{(x-2)(x+2)} + \frac{1}{x-2} \quad \text{M1} \\ &= \frac{-4+x+2}{(x-2)(x+2)} \\ &= \frac{-2+x}{(x-2)(x+2)} \\ &= \frac{1}{x+2} \quad \text{A1} \end{aligned}$$

Answer $\frac{1}{x+2}$... [2]

6 Solve the inequality $x - 5 < \frac{2x}{5} \leq \frac{x}{2} + \frac{1}{5}$.

$$\begin{aligned} x - 5 &< \frac{2x}{5} & \frac{2x}{5} &\leq \frac{x}{2} + \frac{1}{5} \\ 5x - 25 &< 2x & 4x &\leq 5x + 2 \\ 3x &< 25 & x &\geq -2 \quad \text{M1} \\ x &< \frac{25}{3} \text{ or } 8\frac{1}{3} & & \text{M1} \end{aligned}$$

$$-2 \leq x < 8\frac{1}{3} \quad \text{A1}$$

Answer ... $-2 \leq x < 8\frac{1}{3}$ [3]

7 Solve the equation $\left(\frac{1}{5}\right)^{-2} \times 125^x = \sqrt{25^x}$.

$$\left(\frac{1}{5}\right)^{-2} \times 125^x = \sqrt{25^x}$$

$$5^2 \times 5^{3x} = (5^2)^{\frac{x}{2}} \quad \text{M1}$$

$$2 + 3x = x$$

$$2x = -2$$

$$x = -1 \quad \text{A1}$$

Answer $x = \dots -1 \dots [2]$

- 8 (a) Madeline deposited \$6 000 into a 3-month fixed deposit which pays a simple interest. If she receives a total amount of \$6 013.50 at the end of the 3 months, what is the rate of interest per annum?

$$13.50 = \frac{6000(R)\left(\frac{3}{12}\right)}{100} \quad \text{M1}$$

$$R = \frac{13.50 \times 100}{6000 \times \left(\frac{3}{12}\right)}$$

$$= 0.9\% \quad \text{A1}$$

Answer $\dots 0.9 \dots \% [2]$

- (b) Another bank offers an interest rate of 0.6% per annum, compounded monthly. What would be the total amount that Madeline receives, correct to the nearest dollar, at the end of 3 months if she were to put the \$6 000 in this bank?

$$T = 6000 \left(1 + \frac{0.6}{100}\right)^{12\left(\frac{3}{12}\right)} \quad \text{M1}$$

$$= 6009.00$$

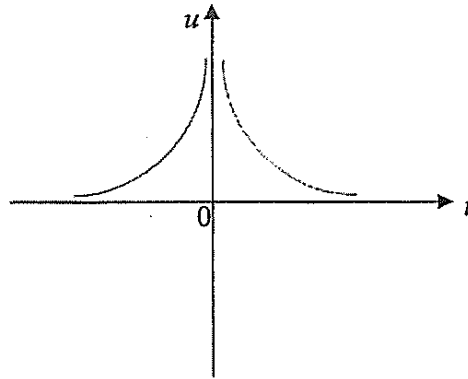
$$= \$6009 \text{ (nearest \$)} \quad \text{A1}$$

Answer \$...6009... [2]

9 u is inversely proportional to the square of t .

(a) Sketch the graph of $u = \frac{2}{t^2}$.

Answer:



[1]

(a) Find the percentage change in u when t is tripled.

$$u = \frac{k}{t^2}$$

$$u_1 = \frac{k}{9t^2} \quad \text{M1}$$

A1

Answer -88.9% [2]

10 These are the first four terms of a sequence.

$$\frac{5}{2} \quad \frac{20}{3} \quad \frac{45}{4} \quad \frac{80}{5}$$

(a) Write down the next two terms.

B2

Answer $\frac{125}{6}$, $\frac{180}{7}$ [1]

(b) Find an expression, in terms of n , for the n th term of this sequence.

Answer $\frac{5n^2}{n+1}$ [1] B1

(c) Explain why $71\frac{3}{16}$ is not a term of this sequence.

Answer $71\frac{3}{16} = \frac{1139}{16}$, since 1139 is not a multiple of 5, $71\frac{3}{16}$ cannot be a term.

.....[1]

- 11 The exchange rate between pounds (£) and Singapore dollars (\$) is £1 = \$1.86.
The exchange rate between Singapore dollars (\$) and euros (€) is \$1 = €0.62.
Megan is comparing day tour packages on a website.

Paris 2-Day Tour €280 per person
London 2-Day Tour £280 per person

Megan claims that the tour package in Paris costs less than the tour package in London.

Is Megan's claims correct? Justify your answer with calculations.

Answer [2]

$$£1 = \$1.86$$

$$€1 = \frac{1}{0.62} = \$1.61 \quad \text{B1}$$

Since it costs less to buy €1 compared to £1, it would be cheaper to go on the tour in Paris. B1

Cost of tour in Paris

$$= \$\frac{280}{0.62}$$

$$= \$451.61 \quad \text{B1}$$

Cost of tour in London

$$= \$1.86 \times 280$$

$$= \$520.80 \quad \text{B1}$$

It costs \$69.19 less in Paris than in London.

- 12 (a) (i) Express 1008 as a product of prime factors, leaving your answer in index notation.

3	1008
3	336
2	112
2	56
7	28
2	4
2	2
	1

$$\text{Answer } 1008 = \dots 2^4 \times 3^2 \times 7 \dots \dots \dots [1]$$

- (ii) p and q are numbers such that q is a prime number.

Find the values of p and q such that $\frac{1008p}{q}$ is the smallest possible cube.

$$\begin{aligned} \frac{1008p}{q} &= \frac{2^4 \times 3^2 \times 7p}{q} \\ &= 2^6 \times 3^3 \\ p &= 2^2 \times 3 = 12 \\ q &= 7 \end{aligned}$$

B2

$$\text{Answer } p = \dots 12 \dots, q = \dots 7 \dots \dots [2]$$

- (b) The lowest common multiple of x and 1008 is $2^5 \times 3^2 \times 5 \times 7$. Find the smallest possible value of x .

$$x = 2^5 \times 5 = 160$$

$$\text{Answer } x = \dots 160 \dots \dots [1]$$

- 13 (a) (i) Express $4 - 2x - x^2$ in the form $a - (x + b)^2$.

$$\begin{aligned} &4 - 2x - x^2 \\ &= -(x^2 + 2x + 1^2 - 4 - 1^2) \\ &= -(x + 1)^2 + 5 \\ &= 5 - (x + 1)^2 \end{aligned}$$

Answer $5 - (x + 1)^2$ [1]

- (ii) Explain, without solving, why there is no solution of x when

$$4 - 2x - x^2 = 6.$$

Answer [1]

The maximum value of $4 - 2x - x^2 = 5$, hence there are no solution of x when $4 - 2x - x^2 = 6$.

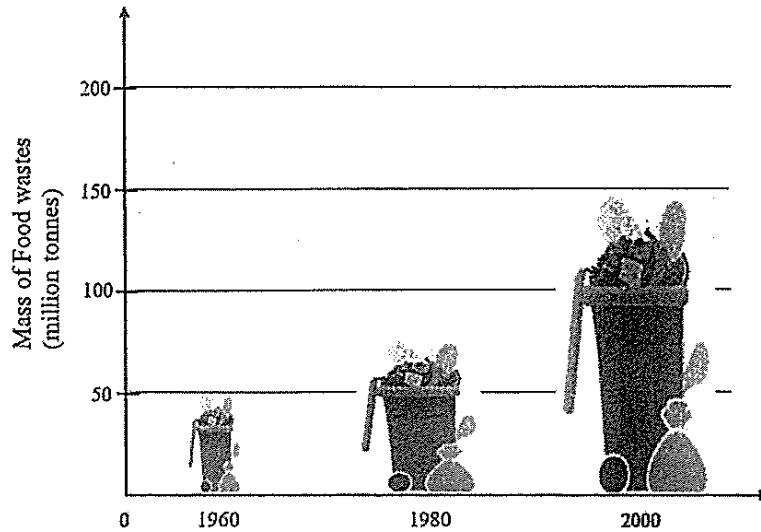
- (b) Using part (a)(i), solve the equation $x^2 + 2x = 8$.

$$\begin{aligned} x^2 + 2x &= 8 \\ -x^2 - 2x &= -8 \\ -x^2 - 2x + 4 &= -4 \\ 5 - (x + 1)^2 &= -4 \quad \text{M1} \\ (x + 1)^2 &= 9 \\ x + 1 &= \pm 3 \\ x &= 2 \text{ or } x = -4 \end{aligned}$$

A2

Answer $x = \dots\dots 2 \dots$ or $\dots\dots -4 \dots\dots$ [3]

14 The graph shows the total mass of food waste collected around a particular neighbourhood at the end of each given year.



<p>(a) State one misleading feature of the graph. <i>Answer [1]</i></p>	<p>(b) Explain how this feature affects the reader's interpretation of the graph. <i>Answer [1]</i></p>
<p>It is not clear if the height or the area of the image is used to determine the mass of food waste collected.</p>	<p>The height and the area of the image is not proportional. The readers may think that there is more than 200% increase in mass from 1960-2000.</p>
<p>The widths of the icons are not the same.</p>	

Not Accepted

- 1) The title is biased.
- 2) The images are of different sizes/inconsistent. → You will need to elaborate further.
- 3) The horizontal axis does not show all the years.

- 15 The following diagram represents the stem-and-leaf scores of sixteen students who took a Mathematics test.

2	1
3	
4	
5	4 5 6 6 7 8 9
6	<i>k</i> 9
7	1 3 5 6 9
8	0

Key: 2 | 1 means 21 marks

- (a) If the median mark is 62, find the possible value of k .

$$\frac{59 + m}{2} = 62$$

$$\begin{aligned} m &= 2(62) - 59 \\ &= 65 \end{aligned}$$

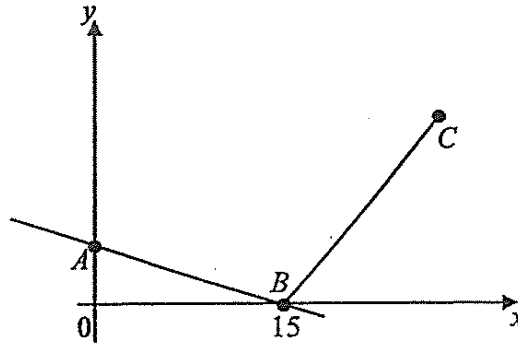
Answer $k = \dots\dots\dots 5 \dots\dots\dots$ [1]

- (b) Explain why the mean may not be appropriate average to use to summarise the results of the Mathematics test.

Answer [1]

The mean is not appropriate as there is an outlier (21 mark) which will decrease the mean /skew the mean value.

- 16 In the diagram, C is the point $(30,15)$, A is a point on the y -axis and B is a point on the x -axis.



- (a) Given that the equation of the line AB is $3y + x = 15$, find the length of AB .

$$A(0,5), B(15,0)$$

$$AB = \sqrt{(15-0)^2 + (0-5)^2}$$

$$= 15.811\dots$$

$$= 15.8 \text{ units}$$

Answer15.8.....units [1]

- (b) Find the equation of the line which passes through $C(30,15)$ and is parallel to the line AB .

$$m = -\frac{1}{3}$$

$$y - 15 = -\frac{1}{3}(x - 30)$$

$$y = -\frac{1}{3}x + 10 + 15$$

$$y = -\frac{1}{3}x + 25$$

Answer $y = -\frac{1}{3}x + 25$ [1]

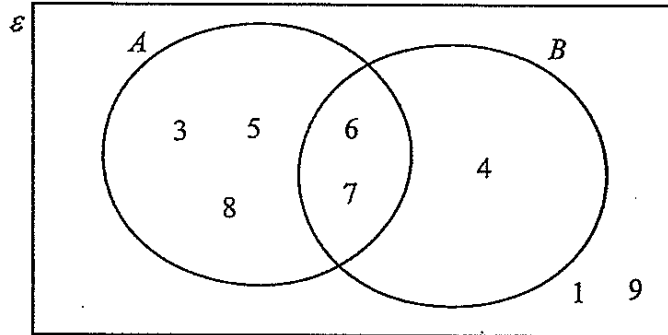
- (c) If $ABCD$ is a parallelogram, find the coordinates of D .

$$D(30-15, 15+5)$$

$$= D(15, 20)$$

Answer D (.....15...,20....) [1]

17 (a)



Use the Venn diagram to answer the following.

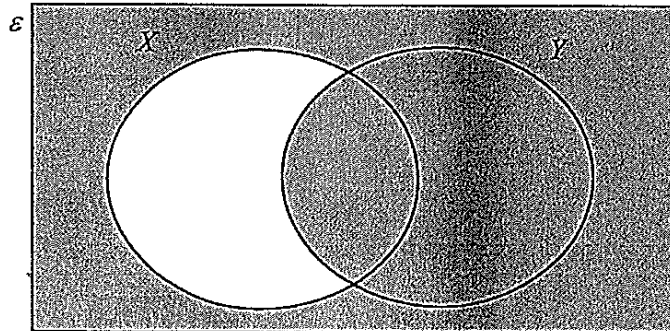
(i) List all the elements in the set $(A \cup B) \cap B'$.

Answer {.....3,5, 8.....} [1]

(ii) Fill in a set notation symbol to complete the statement below.

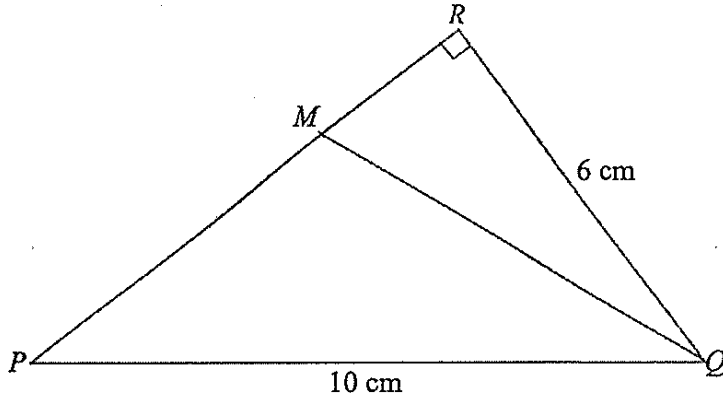
Answer {3, 5, 8} \subset A [1]

(b) Use set notation to describe the shaded region.



Answer $Y \cup X'$ [1]

- 18 Triangle PQR is a right-angled triangle with $PQ = 10$ cm, $QR = 6$ cm and $\angle PRQ = 90^\circ$.
 M is a point on PR such that $11MR = 5PM$.



Find, giving your answer as a fraction in its simplest form, the value of

- (a) $\tan \angle MQR$,

$$\begin{aligned} MR &= \frac{5}{16} PR \\ &= \frac{5}{16} \sqrt{10^2 - 6^2} \\ &= 2.5 \text{ cm} \quad \text{M1} \end{aligned}$$

$$\begin{aligned} \tan \angle MQR &= \frac{2.5}{6} \\ &= \frac{5}{12} \quad \text{A1} \end{aligned}$$

Answer $\frac{5}{12}$ [2]

- (b) $\cos \angle PMQ$,
 $\cos \angle PMQ = -\cos \angle RMQ$

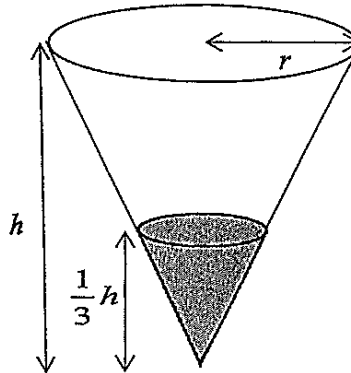
$$\begin{aligned} &= -\frac{2.5}{\sqrt{2.5^2 + 6^2}} \\ &= -\frac{5}{13} \end{aligned}$$

Answer $-\frac{5}{13}$ [1]

- (c) $\frac{\text{area of } \triangle PQM}{\text{area of } \triangle PQR}$

Answer $\frac{11}{16}$ [1]

- 19 The diagram shows an inverted hollow paper cone with a base of radius r cm and a height of h cm. The cone is filled with water, represented by the shaded region, to a depth of $\frac{1}{3}h$ cm.



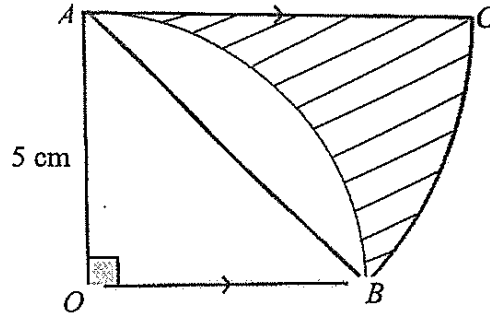
The internal surface area of the cone is 106 cm^2 .

Find the surface area of the inside of the cone that is in contact with the water.

$$\begin{aligned} \frac{106}{A_2} &= \left(\frac{3}{1}\right)^2 && \text{M1} \\ A_2 &= \frac{106}{9} \\ &= 11.7777\dots \\ &= 11.8 \text{ cm}^2 \text{ (3sf)} && \text{A1} \end{aligned}$$

Answer11.8..... cm^2 [2]

- 20 AB is an arc of a circle with the centre O , $\angle AOB = \frac{\pi}{2}$ radians and $OA = 5$ cm.
 AC is parallel to OB and BC is an arc of a circle with centre A .



- (a) Find the length of arc BC .

$$AB = \sqrt{5^2 + 5^2} = 5\sqrt{2} \text{ cm}$$

M1 for either AB or $\angle CAB$

$$\angle CAB = \frac{\pi}{4} \text{ radians}$$

$$BC = \sqrt{50} \left(\frac{\pi}{4} \right) \quad \text{M1}$$

$$= 5.5536\dots$$

$$= 5.55 \text{ cm (3sf)} \quad \text{A1}$$

Answer5.55..... cm [3]

- (b) Find the percentage of the figure $OACB$ that is **not** shaded.

$$\text{Area of } OACB = \frac{1}{2}(5)(5) + \frac{1}{2}(\sqrt{50})^2 \left(\frac{\pi}{4} \right) = 32.1349\dots \text{ cm}^2 \quad \text{M1}$$

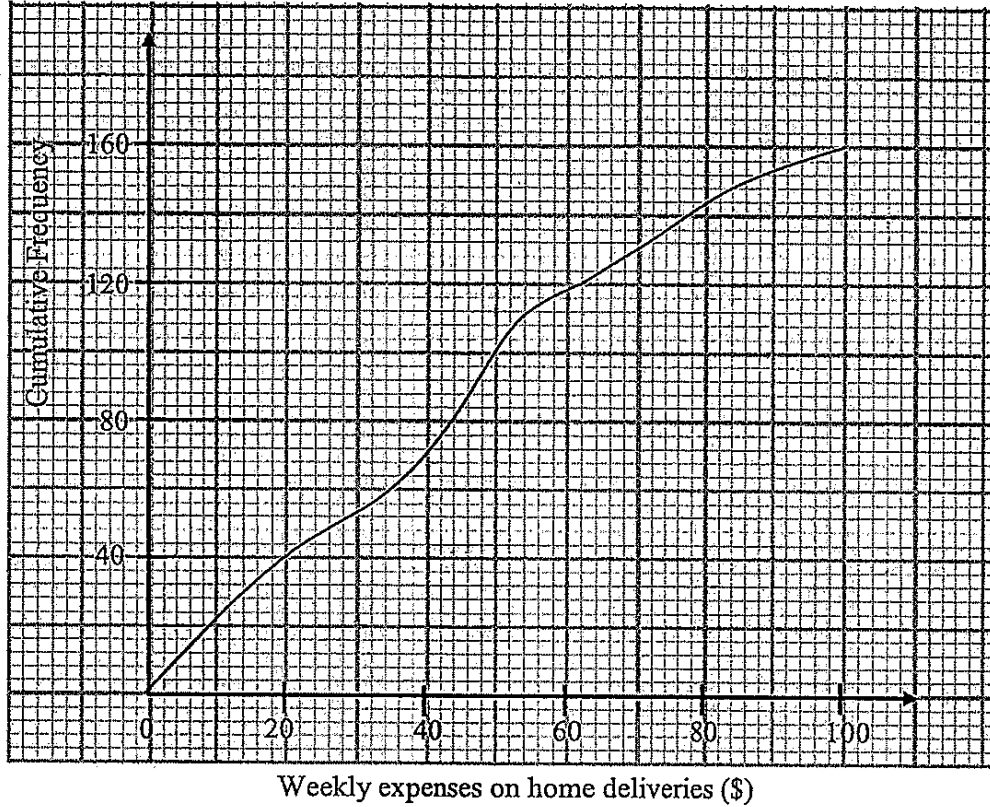
$$\text{Percentage of figure not shaded} = \frac{\frac{1}{4}\pi(5)^2}{32.1349\dots} \times 100\% \quad \text{M1}$$

$$= 61.1016\dots\%$$

$$= 61.1\% \text{ (3sf)} \quad \text{A1}$$

Answer61.1.....% [3]

- 21 The cumulative frequency curve shows the weekly expenditure on home deliveries of 160 households from Block W.



- (a) Use the cumulative frequency curve to find

- (i) the median of the distribution,

Answer \$.....44..... [1]

- (ii) the interquartile range of the distribution,

Interquartile range = \$ (62 – 20) = \$42

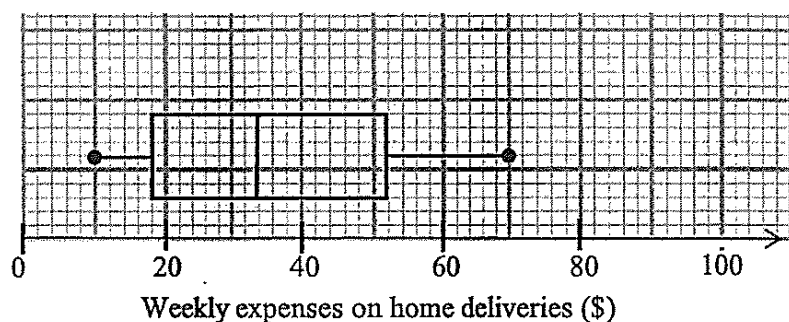
Answer \$.....42..... [1]

- (iii) the value of p , given that $p\%$ of the residents spent more than \$84 weekly on home deliveries.

$$p = \frac{160 - 148}{160} \times 100\% = 7.5\%$$

Answer $p = \dots\dots\dots 7.5 \dots\dots\dots$ [1]

- (b) The weekly expenditure on home deliveries of a group of 160 residents from Block K are displayed in the box and whisker plot below.



Here are two statements comparing the weekly expenditure on home deliveries of the residents from Block W and Block K .

For each statement, state whether you agree or disagree.

Give a reason for each answer, stating clearly which statistic you used to make your decision.

- (i) On average, the residents in Block W has lower weekly expenses on home deliveries than the residents in Block K .

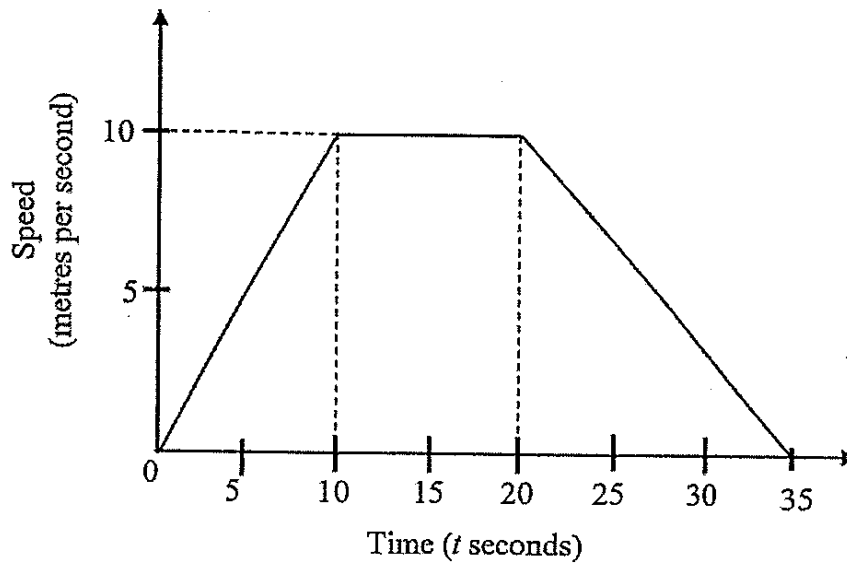
Answer

I **DISAGREE** because residents in Block W has a higher median expenditure of \$44 than the residents in Block K (\$34) [1]

- (ii) The weekly expenditure on home deliveries of the residents in Block K is closer to the median than the weekly expenditure on home deliveries of the residents from Block W .

Answer I **AGREE** because the interquartile range of Block K (\$36) is lower than the interquartile range Block W (\$42) showing that the data is less widely spread than Block W 's [1]

22



The speed-time graph of a particle is shown in the diagram above.

(a) Find

(i) the speed when $t = 25$,

$(35,0), (20,10), (25, v)$

$$\frac{v-0}{25-35} = \frac{10-0}{20-35} \quad \text{M1}$$

$$v = \frac{-10(10)}{-15}$$

$$= 6.67 \text{ (3sf)} \quad \text{A1}$$

Answer 6.67m/s [2]

(ii) the distance covered in the first 35 seconds.

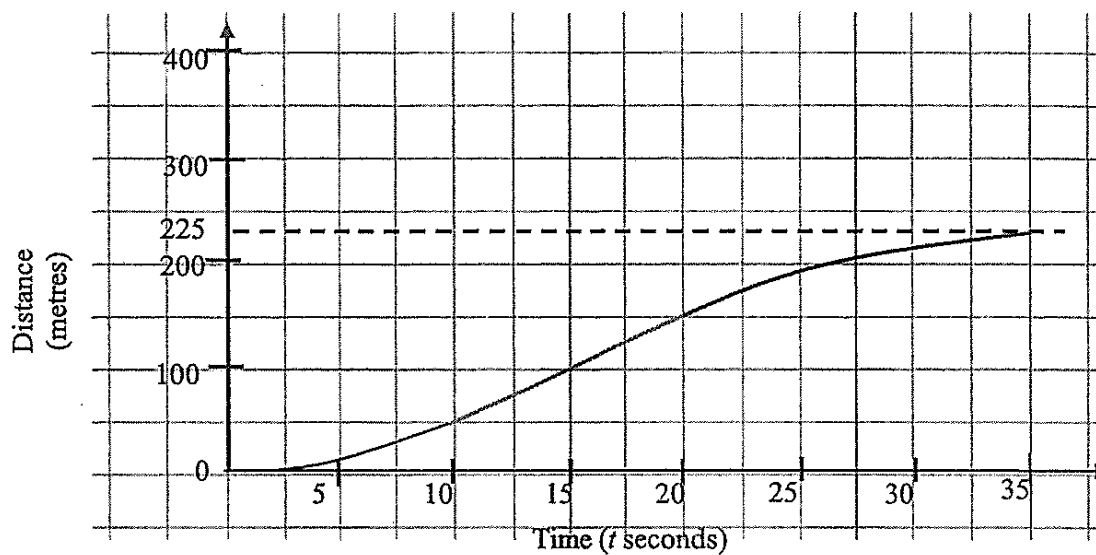
$$\text{Dist} = \frac{1}{2}(10)(10) + 10(10) + \frac{1}{2}(10)(15) \quad \text{M1}$$

$$= 50 + 100 + 75$$

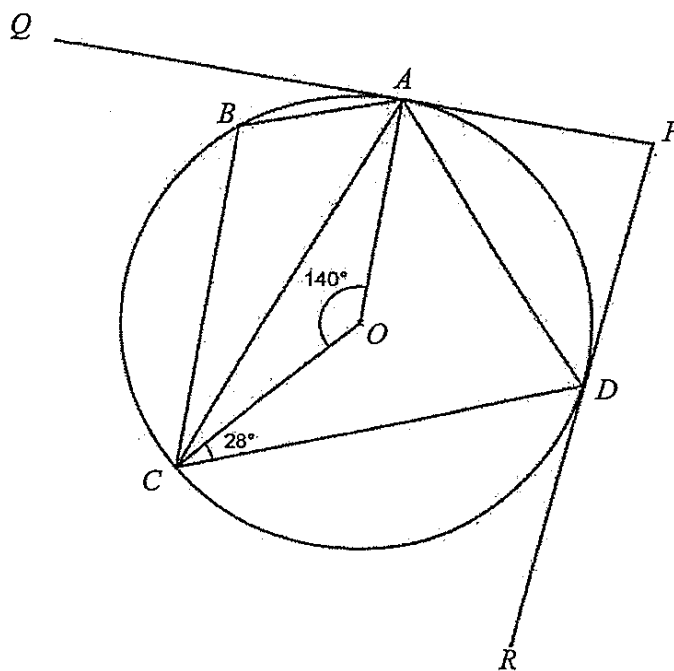
$$= 225 \text{ m} \quad \text{A1}$$

Answer 225m [2]

- (b) Part of the distance-time graph for the same particle is shown in the answer space.
Complete this graph. [1]



- 23 In the diagram, A, B, C and D lie on a circle, centre O .
 PQ and PR are tangents to the circle at A and D respectively.
 It is given that $\angle AOC = 140^\circ$, $\angle DCO = 28^\circ$ and OA is parallel to CB .



- (a) Show, stating your reasons clearly, that CA bisects $\angle BCO$.

Answer [2]

$$\angle OCA = \angle OAC \text{ (base } \angle \text{ of isos. } \Delta)$$

$$= \frac{180^\circ - 140^\circ}{2}$$

$$= 20^\circ \quad \text{M1}$$

$$\angle BCA = \angle OAC \text{ (alt. } \angle \text{s) or } \angle BCA = 180^\circ - 140^\circ - 20^\circ \text{ (int. } \angle)$$

$$= 20^\circ \quad \text{M1}$$

Since $\angle OCA = \angle BCA = 20^\circ$,

CA bisects $\angle BCO$.

(b) Find, stating your reasons clearly,

(i) $\angle BAC$,

Reflex $\angle AOC = 360^\circ - 140^\circ$ (\angle s at a point)

$$= 220^\circ$$

$$\angle ABC = \frac{220^\circ}{2} \quad (\angle \text{ at centre} = 2\angle \text{ at circumference})$$

$$= 110^\circ \quad \text{M1}$$

$$\angle BAC = 180^\circ - 20^\circ - 110^\circ \quad (\angle \text{ s sum of } \Delta)$$

$$= 50^\circ \quad \text{A1}$$

Answer $\angle BAC = \dots\dots\dots 50 \dots\dots^\circ$ [2]

(ii) $\angle PDA$,

$\angle PDA = 20^\circ + 28^\circ$ (\angle in alt segment)

$$= 48^\circ$$

OR

$\angle OAP = 90^\circ$ (radius \perp tangent)

$\angle OAD = 180^\circ - 20^\circ - 20^\circ - 28^\circ - 50^\circ - 20^\circ$ (\angle in opp segment) M1

$$= 42^\circ$$

$\angle PAD = 90^\circ - 42^\circ$

$$= 48^\circ$$

$= \angle PDA$ (base \angle of isos. Δ) A1

Answer $\angle PDA = \dots\dots\dots 48 \dots\dots^\circ$ [2]

(c) Lizzy claims that $QADR\dots$ forms a part of a regular n -sided polygon.

Justify, with reasons, if you agree or disagree with Lizzy.

Answer [1]

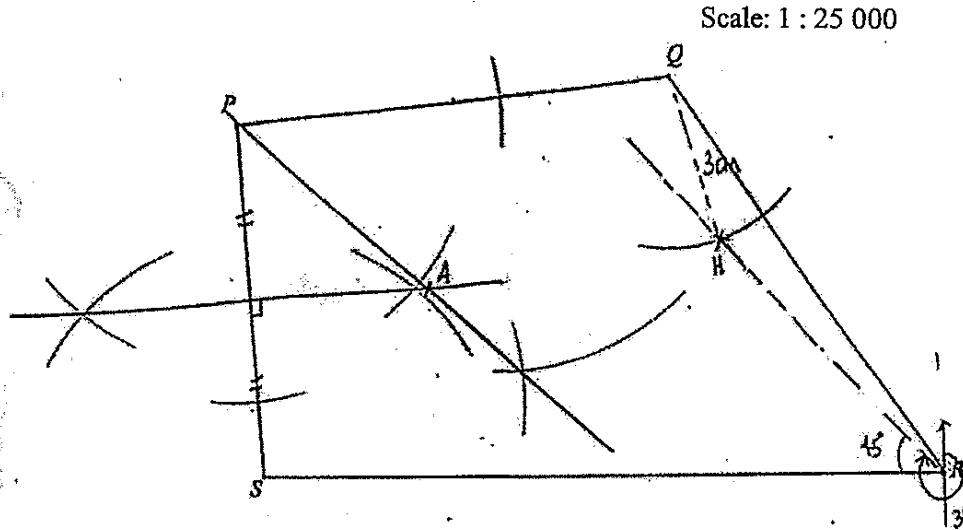
ext. \angle of polygon $\angle PAD = 48^\circ$

$$\text{no. of sides} = \frac{360}{48} = 7.5$$

Since the number of sides is not an integer, $QADR$ is not a regular polygon

Hence I disagree with Lizzy.

- 24 On the scale drawing, P, Q, R and S are the positions of four garden statues in a park. $PQRS$ is a quadrilateral.



- (a) A water-cooler is to be built at A where it is equidistant from the line segments PS and PQ and the points P and S .

On the scale drawing, mark the point A .

[2]

- (b) The garden statue at R is due east of S .
The gardener plans to build a pavilion at H .
The bearing of H from R is 315° and 750 m from Q .

- (i) Calculate the distance HQ on the scale drawing.

Scale:

1 cm : 25 000 cm

1 cm : 250 m

3 cm : 750 m

Answer $HQ = \dots\dots\dots 3 \dots\dots\dots$ cm [1]

- (ii) Mark the point H on the diagram above.

[1]

~ End of Paper ~