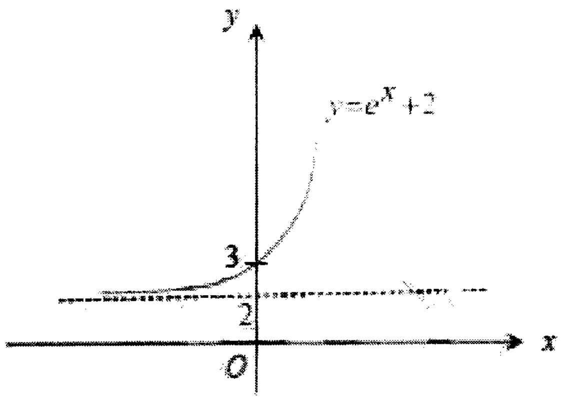
Marking Scheme for A Maths Paper 2

Qn	Working	Marks	Remarks
1(a)		B1 B1	Shape of the curve *y*-intercept and horizontal asymptote
(b)	$3-e^{-x}=2e^{x}$		
	$3-\frac{1}{e^{x}}=2e^{x}$		
	$3e^{x}-1=2e^{2x}$		
	$2e^{2x}-3e^{x}+1=0$	M1	
	Let $y=e^{x}$		
	$2y^{2}-3y+1=0$		
	$(2y-1)(y-1)=0$	M1	
	$y=\frac{1}{2}$ or $y=1$		
	$e^{x}=\frac{1}{2}$ or $e^{x}=1$	M1	
	$x=\ln\frac{1}{2}$ or $x=\ln 1$		
	$x=-0.693$ or $x=0$	A1	

2(i)	Since roots of $f(x) = 0$ are 1, k and k^2 $f(x) = -(x-1)(x-k)(x-k^2)$ $f(2) = -7$ $-(2-1)(2-k)(2-k^2) = -7$ $(2-k)(2-k^2) = 7$ $4-2k^2-2k+k^3 = 7$ $k^3+2k^2+2k-3 = 0$ (shown)	M1 M1 A1	
(ii)	Let $g(k) = k^3-2k^2-2k-3$ $g(3) = 3^3-2(3)^2-2(3)-3$ $= 0$ Since $g(3) = 0$, $k-3$ is a factor of $g(k)$. $k^3-2k^2-2k-3 = 0$ $(k-3)(k^2+k+1) = 0$ $k^2+k+1 = 0$ $k = 3$ or $b^2-4ac = 1^2-4(1)(1)$ $= -3 < 0$ (no real roots) Therefore $k^3-2k^2-2k-3 = 0$ has only 1 real root.	M1 M1 M1, M1 M1 B1	

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3(i)	$y=(x+2)\sqrt{x-1}$ $\frac{dy}{dx}=(x+2)[\frac{1}{2}(x-1)^{-\frac{1}{2}}]+\sqrt{x-1}(1)$ $=(x-1)^{-\frac{1}{2}}[\frac{1}{2}x+1+x-1]$ $=(x-1)^{-\frac{1}{2}}(\frac{3}{2}x)$ $=\frac{3x}{2\sqrt{x-1}}$	M1 M1 M1 A1	
(ii)	By Chain Rule, $\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$ $2=\frac{3x}{2\sqrt{x-1}}\times\frac{dx}{dt}$ $\frac{dx}{dt}=2\div\frac{3(2)}{2\sqrt{2-1}}$ $=\frac{2}{3}$ units/s	M1 A1	
(iii)	$\int_2^5\frac{x}{\sqrt{x-1}}\,dx=\frac{2}{3}\int_2^5\frac{3x}{2\sqrt{x-1}}\,dx$ $=\frac{2}{3}[(x+2)\sqrt{x-1}]_2^5$ $=\frac{2}{3}[7\sqrt{4}-4\sqrt{1}]$ $=\frac{2}{3}(10)$ $=6\frac{2}{3}$	M1 M1 A1	

4(i)	Let E be the point on AB such that BE is perpendicular to CE. F is the point on CE such that CF is perpendicular to FD. $\cos\theta = \frac{BE}{4}$ $BE = 4\cos\theta$ $\sin\theta = \frac{FD}{1}$ $FD = \sin\theta$ $AB = BE + FD$ $= 4\cos\theta + \sin\theta$ (shown)	M1 for either one correct A1	
(ii)	$AB = \sqrt{4^2+1^2}\cos\left(\theta - \tan^{-1}\frac{1}{4}\right)$ $= \sqrt{17}\cos(\theta - 14.036°)$ $= \sqrt{17}\cos(\theta - 14.0°)$	M1 for finding R, M1 for finding α A1	
(iii)	Max $AB = \sqrt{17}$ m For AB to be maximum, the value of $\cos(\theta - 14.036°)$ must be 1. $\cos(\theta - 14.036°) = 1$ $\theta - 14.036° = 0°$ $\theta = 14.036°$ $= 14.0°$ (1 d.p.)	B1 B1	
(iv)	$3 = \sqrt{17}\cos(\theta - 14.036°)$ $\cos(\theta - 14.036°) = \frac{3}{\sqrt{17}}$ $\theta - 14.036° = 43.313°$ $\theta = 43.313° + 14.036°$ $= 57.349°$ $= 57.3°$ (1 d.p.)		

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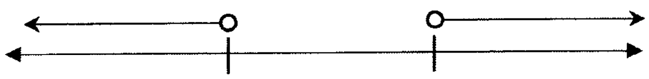
5(a)	$y=2x^2-6x+c$ --------- (1) $y+2x=8$ --------------- (2) Equating (1) & (2): $2x^2-6x+c=8-2x$ $2x^2-4x+c-8=0$ The line is a tangent to the curve, $b^2-4ac=0$ $(-4)^2-4(2)(c-8)=0$ $16-8c+64=0$ $c=10$	 M1 M1 A1	
(b)	$3(x^2-5)>x-1$ $3x^2-15-x+1>0$ $3x^2-x-14>0$ $(3x-7)(x+2)>0$ $x<-2$ or $x>2\frac{1}{3}$ 	 M1 A1 A1	
(c)	$-2x^2+x-p$ $a=-2,\ b=1,\ c=-p$ For real roots, $b^2-4ac\geq 0$ $1-4(-2)(-p)\geq 0$ $1-8p\geq 0$ $p\leq\frac{1}{8}$ Greatest value of integer $p=0$	 M1 M1 A1	

6(i)	$\left(\frac{1}{2}-2x\right)^5$ $=\left(\frac{1}{2}\right)^5+\binom{5}{1}\left(\frac{1}{2}\right)^4(-2x)^1+\binom{5}{2}\left(\frac{1}{2}\right)^3(-2x)^2+\binom{5}{3}\left(\frac{1}{2}\right)^2(-2x)^3+$ $=\frac{1}{32}-\frac{5x}{8}+5x^2-20x^3+$	M1 A1	
(ii)	$(1+ax+3x^2)\left(\frac{1}{2}-2x\right)^5$ $=(1+ax+3x^2)\left(\frac{1}{32}-\frac{5x}{8}+5x^2-20x^3+...\right)$ Considering x^2, $5x^2-\frac{5}{8}ax^2+\frac{3}{32}x^2=\frac{13}{2}x^2$ $5-\frac{5}{8}a+\frac{3}{32}=\frac{13}{2}$ $a=-2\frac{1}{4}$	M1 M1 M1 A1	
(iii)	$(0.47)^5=\left(\frac{1}{2}-2x\right)^5$ $\frac{1}{2}-2x=0.47$ $x=0.015$ $(0.47)^5=\frac{1}{32}-\frac{5(0.015)}{8}+5(0.015)^2-20(0.015)^3+...$ ≈ 0.02293	M1 M1 A1	

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<table>
<tr><td>7(i)</td><td>

$y=10+Ab^x$
$y-10=Ab^x$
Take log to the base 10 on both sides,
$\lg(y-10)=\lg b(x)+\lg A$
Plot $\lg(y-10)$ against x to obtain a straight line.

<table>
<tr><td>x</td><td>0.5</td><td>1.0</td><td>1.5</td><td>2.0</td></tr>
<tr><td>y</td><td>15.9</td><td>19.1</td><td>23.4</td><td>30.2</td></tr>
<tr><td>$y-10$</td><td>5.9</td><td>9.1</td><td>13.4</td><td>20.2</td></tr>
<tr><td>$\lg(y-10)$</td><td>0.77</td><td>0.96</td><td>1.13</td><td>1.31</td></tr>
</table>

</td><td>M1, G1</td><td>1 mark for drawing the straight line</td></tr>
<tr><td>(ii)</td><td>

$\lg A=$ vertical intercept of the graph
$=0.60$ (Accept ± 0.02)
$A=10^{0.60}$
≈ 3.98

$\lg b=$ Gradient of the graph
$=\dfrac{1.31-0.60}{2-0}$
$=0.355$ (Accept ± 0.02)
$b=10^{0.355}$
≈ 2.26

</td><td>M1

A1

M1

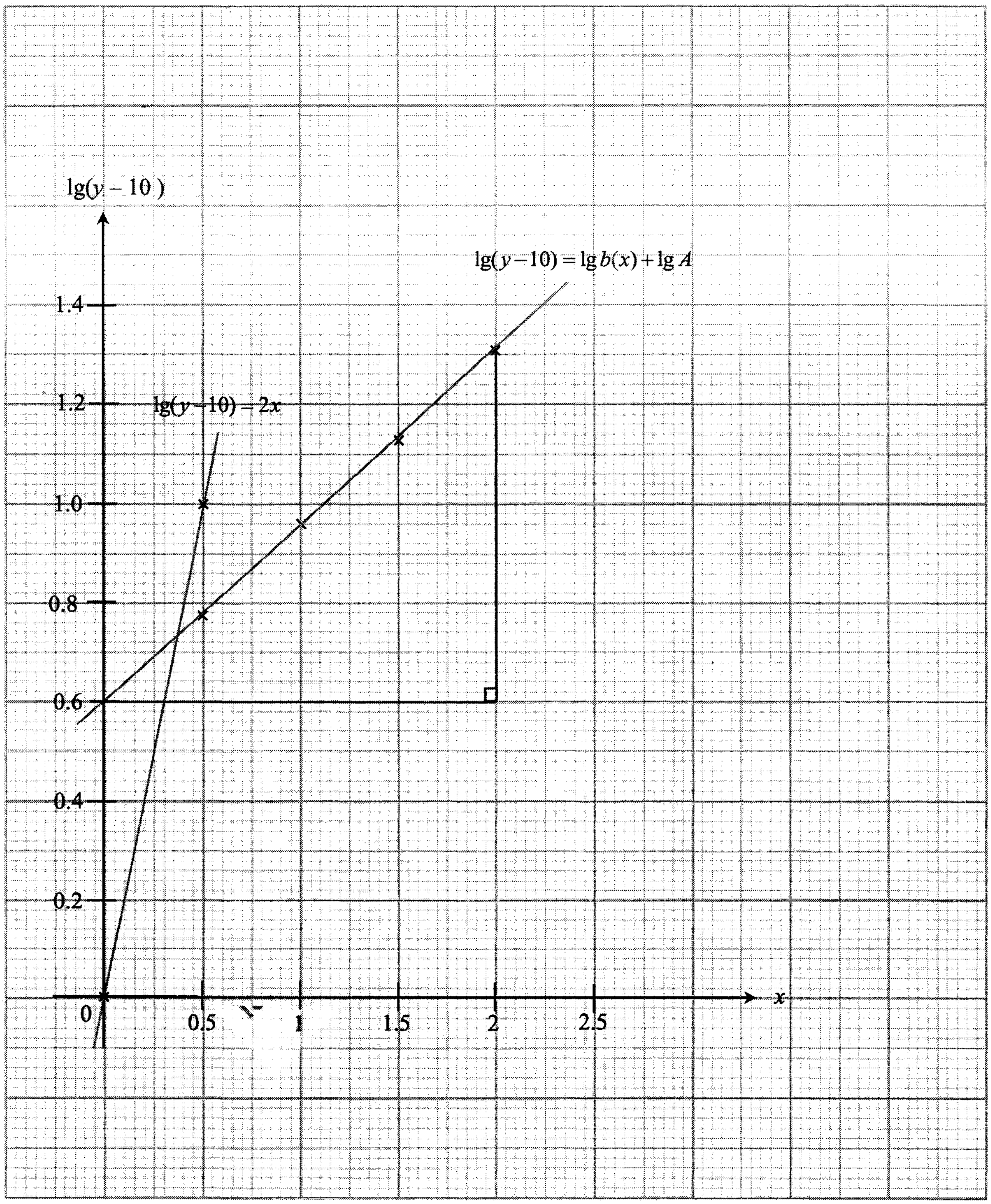
A1</td><td></td></tr>
<tr><td>(iii)</td><td>

$Ab^x=10^{2x}$
$\lg A+x\lg b=2x$
$\therefore$ draw $\lg(y-10)=2x$

From the graph, $x\approx 0.375$ (Accept ± 0.01)

</td><td>M1
M1

A1</td><td></td></tr>
</table>



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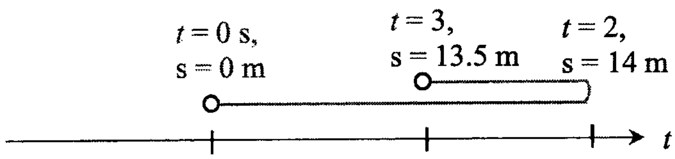
8(i)	$v = 3t^2 + kt + 18$ $a = \frac{dv}{dt}$ $= 6t + k$ When $t = 1$, $6(1) + k = -9$ $k = -15$ (shown)	M1 A1	
(ii)	$3t^2 - 15t + 18 = 0$ $t^2 - 5t + 6 = 0$ $(t-2)(t-3) = 0$ $t = 2$ or $t = 3$	M1 A1 for both answers	
(iii)	$s = \int v \, dt$ $= \int (3t^2 - 15t + 18) \, dt$ $= t^3 - \frac{15}{2}t^2 + 18t + C$ When $t = 0, s = 0, C = 0$ $\therefore s = t^3 - \frac{15}{2}t^2 + 18t$ when $t = 2$ s, $s = 14$ m when $t = 3$ s, $s = 13.5$ m  Distance travelled in first 3 s = 14 + (14 – 13.5) = 14.5 m	 M1 M1 M1 A1	

9(i)	when $y = 0$, $4 - e^{-2x} = 0$ $e^{-2x} = 4$ $-2x = \ln 4$ $x = -\ln 2$ $\therefore A(-\ln 2, 0)$ when $x = 0$, $y = 4 - e^{0}$ $= 3$ $\therefore B(0,3)$	B1 B1	
(ii)	$y = 4 - e^{-2x}$ $\frac{dy}{dx} = -(-2)e^{-2x}$ $= 2e^{-2x}$ when $x = 0$, $\frac{dy}{dx} = 2e^{0}$ $= 2$ Gradient of $BC = -\frac{1}{2}$ Let the coordinates of C be (x, y). $\frac{3-0}{0-c} = -\frac{1}{2}$ $c = 6$ $\therefore C(6,0)$	M1 M1 M1 A1	
(iii)	Area of shaded region $= \int_{-\ln 2}^{0} (4 - e^{-2x})\ dx + \frac{1}{2}(6)(3)$ $= \left[4x + \frac{1}{2}e^{-2x}\right]_{-\ln 2}^{0} + 9$ $= \left(0 + \frac{1}{2}\right) - \left(-4\ln 2 + \frac{1}{2}e^{2\ln 2}\right) + 9$ $= 7\frac{1}{2} + 4\ln 2$ ≈ 10.3 units^2	M1, M1 M1 M1 A1	

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10(i)	$(x-3)^2+(y+2)^2=12+9+4$ $(x-3)^2+(y+2)^2=25$ $C(3,-2)$ Radius $=\sqrt{25}$ $=5$ units	B1 B1	
(ii)	Normal of a circle passes through centre of a circle. $3(-2)=m-4(3)$ $m=6$	M1 A1	
(iii)	$3y=6-4x$ $y=-\frac{4}{3}x+2$ --------------- (1) $(x-3)^2+(y+2)^2=25$ --------------- (2) $(x-3)^2+(-\frac{4}{3}x+2+2)^2=25$ $(x-3)^2+(4-\frac{4}{3}x)^2=25$ $x^2-6x+9+(\frac{16}{9}x^2-\frac{32}{3}x+16)=25$ $\frac{25}{9}x^2-\frac{50}{3}x=0$ $25x^2-150x=0$ $25x(x-6)=0$ $x=6$ (rejected) or $x=0$ when $x=0$, $y=-\frac{4}{3}(0)+2$ $y=2$ $\therefore A(0,2)$ (shown)	M1 M1 A1 A1	
(iv)	$(3,-2)$ is midpoint of AB and the required coordinates B be $B(e,f)$ $(\frac{0+e}{2},\frac{2+f}{2})=(3,-2)$ $B(6,-6)$ Equation of tangent at B: $y+6=\frac{3}{4}(x-6)$ $y=\frac{3}{4}x-\frac{21}{2}$	M1 M1 A1	