MARKING SCHEME FOR PAPER 1

1 (a) Evaluate $4.9\times10^6 - 5.64\times10^4$.

Give your answer in standard form, correct to 2 significant figures.

Answer …… 4.8×10^6 ……………… [1]

(b) 1.37×10^{-7} seconds can be written as k nanoseconds.

Find the value of k.

(1 nanosecond $=1\times10^{-9}$ seconds)

$$k\times10^{-9}=1.37\times10^{-7}$$

$$k=\frac{1.37\times10^{-7}}{10^{-9}}$$

$$=137$$

Answer ………137………. [1]

2 Given that $0°\le\theta\le180°$, find the value(s) of θ for $\sin\theta=0.772$

$\sin\theta=0.772$

$\theta=50.53$ or $\theta=180-50.53$

$=50.5$ $\quad=129.47$

Answer $\theta=$ ..50.5^0… or .. 129.5^0… [2]

3 Find the value of p in $3^{25}-3^{24}=2\times3^{p+1}$.

$3^{25}-3^{24}=2\times3^{p+1}$

$3^{24}(3-1)=2\times3^{p+1}$ [M1]

$2\times3^{24}=2\times3^{p+1}$

$p+1=24$

Answer $p=$ …23……[A1]……..… [2]

4 (a) Express 1008 as a product of its prime factors.

2	1008
2	504
2	252
2	126
3	63
3	21
7	7
	1

$1008=2^4\times3^2\times7$

Answer …… $2^4\times3^2\times7$ …………….. [1]

(b) Hence, find the smallest positive integer k such that $1008k$ is a perfect cube.

$k=2^2\times3\times7^2=588$

Answer ………588……………..……. [1]

5 In country X, the Goods and Services Tax (GST) is predicted to rise from 7% to 8.5% in the future. Before the increase, the price of a tablet is $1391 including 7% GST. Calculate the new price inclusive of GST, after the GST has increased to 8.5%.

Price of tablet = $1300

Price including 8.5% GST = $1 410.50 [Answer must be given in 2 dp] OR

Price including 8.5% GST $=\frac{108.5}{107}\times1391$

$=\$1\,410.50$ [Answer must be given in 2 dp]

Answer $…1 410.50……..…. [2]

6 Simplify $m^{\frac{3}{2}}n^{-5} \div \left(\frac{n}{m}\right)^{-2}$, giving your answer in positive index notation.

$$m^{\frac{3}{2}}n^{-5} \div \left(\frac{n}{m}\right)^{-2} = m^{\frac{3}{2}}n^{-5} \times \left(\frac{n}{m}\right)^{2}$$

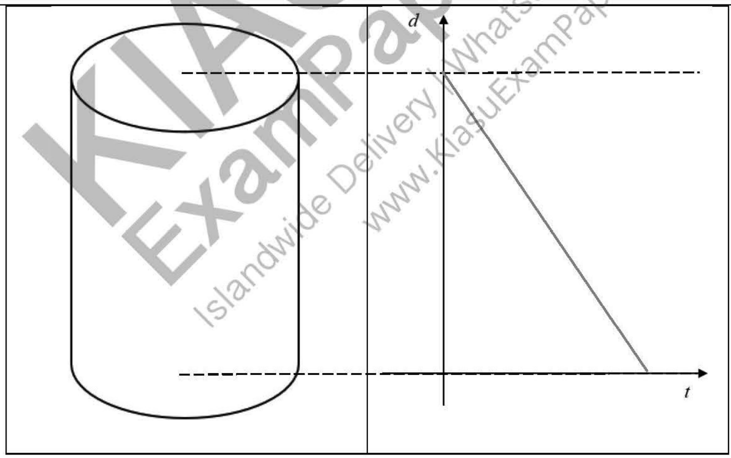
$$= m^{\frac{3}{2}-2}n^{-5+2}$$

$$= m^{-\frac{1}{2}}n^{-3}$$

$$= \frac{1}{m^{\frac{1}{2}}n^{3}}$$

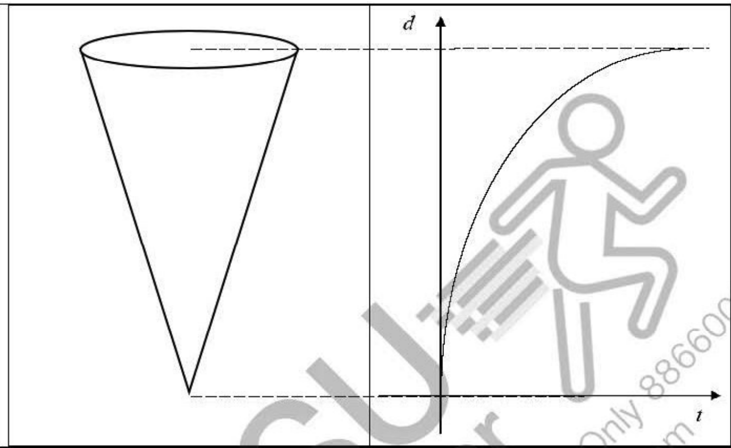
Answer ………………………………… [3]

7 **(a)** Liquid is **flowing out of the full container** at a constant rate. After t seconds, the depth of water in the container is d cm. Sketch the graph of d against t.



[1]

(b) Liquid is **poured into the empty container** at a constant rate. After t seconds, the depth of water in the container is d cm. Sketch the graph of d against t.



[1]

8 Liquid detergent is sold in two sizes.

Which container is better value for money? [3]

Explain your answer.

Answer

Small bottle:

Unit Cost = $\frac{475}{306} = 1.55$ cents/ml

Large bottle:

Unit Cost = $\frac{1371}{1360} = 1.01$ cents/ml

The large bottle is better value for money because it costs 1.01 cents per ml (or \$0.0101 per ml) instead of 1.55 cents per ml (or \$0.0155 per ml).

OR

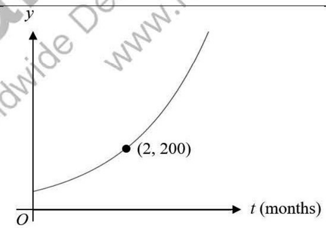
Small bottle: \$4.75 for 306 ml

Big bottle: $\frac{13.71}{1360} \times 306 = \$3.08$ for 306 ml

The large bottle is better value for money because it costs less for the same amount of detergent.

9	(a)	Solve the inequalities $3x-1<2x+9\le\frac{1}{2}(7x-6)$.	
		$3x-1<2x+9$ $x<10$ AND $2x+9\le\frac{1}{2}(7x-6)$ $4x+18\le 7x-6$ $-3x\le -24$ $x\ge 8$ $8\le x<10$ — Deduct the answer mark if OR is used.	
	(b)	Show your solution in **(a)** on the number line below.	
			[B1]

10	A map is drawn to a scale of 1 cm to 4 km.			
	(a)	Express the scale of the map in the form of 1 : *n*.		
		Answer	……… 1 : 400 000 ………	[1]
	(b)	The distance between two schools is 8.4 km. Find the corresponding distance, in centimetres, on the map.		
		$\frac{8.4}{4}=2.1$ cm		
		Answer	…………2.1………… cm	[1]
	(c)	One of the schools occupies an area of 2.5 cm^2 on the map. Calculate its actual area in square kilometres.		
		1 cm rep. 4 km 1 cm^2 rep 16 km^2 2.5 cm^2 rep 40 km^2		
		Answer	…………40………… km^2	[2]

11	**(a)** The number of a certain type of insects, *y*, on an island *t* months after the first observation is given by $y=k(2)^t$, where *k* is a positive constant. The point (2, 200) lies on the graph of $y=k(2)^t$		
			
	(i) Find the value of *k*.		
	$200=k(2)^2$ $k=50$		

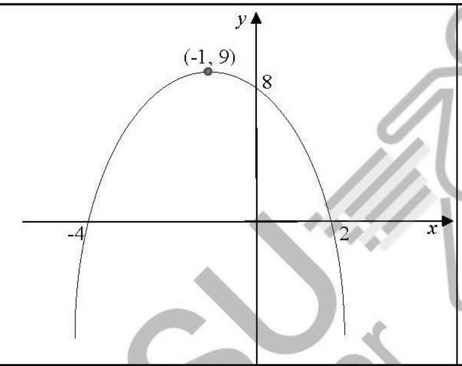
Answer $k =$ ………50……………. [1]

(ii) Find the initial value of y.

$y = 50(2)^0$

$y = 50$

(b) Sketch the graph of $y = -(x+1)^2 + 9$.



B1 for shape of curve.

B1 for Turning point.

B1 for x-and y-intercepts.

12 Two jars are geometrically similar. Their heights are 8 cm and 14 cm respectively.

(a) Calculate the volume of the smaller jar as a percentage of the volume of the larger jar. Give your answer correct to the nearest whole number.

$\frac{V_1}{V_2} = \left(\frac{8}{14}\right)^3 = \frac{64}{343}$ [M1]

Percentage $= \frac{64}{343} \times 100\% = 18.65889\%$

$= 19\ \%$ [A1]

(b) The base area of of the larger jar is 140 cm².

Find the base area of the smaller jar.

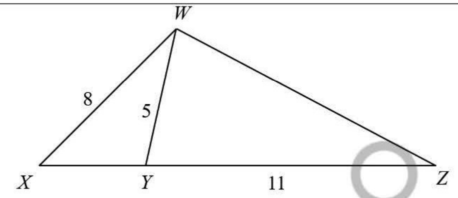
$\frac{A_S}{A_L} = \left(\frac{h_S}{h_L}\right)^2$

$\frac{A_S}{140} = \left(\frac{8}{14}\right)^2$

$A_s = \left(\frac{8}{14}\right)^2 \times 140$

$= 45.7$ or $45\frac{5}{7}$ cm²

13



In triangle WXZ, $WX = 8$ cm.

Y is a point on XZ such that $WY = 5$ cm, $YZ = 11$ cm and $\sin\angle WYZ = \frac{4}{5}$.

(a) Find the area of triangle WYZ.

Area of triangle $WYZ = \frac{1}{2} \times 5 \times 11 \times \frac{4}{5}$

$= 22$ cm²

Answer …………22……………… [2]

(b) Find the exact size of angle WXY.

$\frac{\sin\angle WXY}{5} = \frac{\sin\angle WYX}{8}$

$\frac{\sin\angle WXY}{5} = \frac{\sin\angle WYZ}{8}$

$\sin\angle WXY = \frac{5 \times \frac{4}{5}}{8}$

$\angle WXY = 30°$

Answer …………30^0……………. [2]

14 Factorise completely

(a) $x^2 - y^2 - 3x + 3y$,

$x^2 - y^2 - 3x + 3y$

$= (x+y)(x-y) - 3(x-y)$

$= (x-y)(x+y-3)$

(b) $32a^2 - 50$

$32a^2 - 50$
$= 2(16a^2 - 25)$
$= 2(4a+5)(4a-5)$

15 The nth term of a sequence is given by $T_n = n^2 + 3$.

(a) Explain why 1010 is not a term in the sequence.

Answer:

If $1010 = n^2 + 3$, then $n = \sqrt{1007} = 31.7$ (3 sf), which is not a positive integer. [B1]

(b) Find an expression, in terms of n, for $T_{n+1} - T_n$.

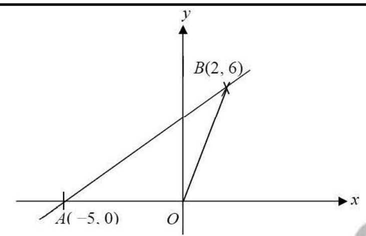
$T_{n+1} - T_n = (n+1)^2 + 3 - [n^2 + 3]$
$= n^2 + 2n + 1 + 3 - n^2 - 3$
$= 2n + 1.$

Answer ………$2n+1$……………. [2]

(c) Given that the difference between two successive terms in the sequence is 99, use your answer in part **(b)** to find the smaller term.

$2n + 1 = 99$
$n = 49$
$T_{49} = 49^2 + 3$
$= 2404.$

16 In the diagram, a straight line passes through the points $A(-5, 0)$ and $B(2, 6)$.



(a) Find the equation of the line AB,

$Gradient = \frac{6-0}{2-(-5)} = \frac{6}{7}$ $y - 6 = \frac{6}{7}(x-2)$ $y = \frac{6}{7}x + \frac{30}{7}$ $7y = 6x + 30$	OR	$Gradient = \frac{6-0}{2-(-5)} = \frac{6}{7}$ Sub (2, 6) into $y = \frac{6}{7}x + c$ $6 = \frac{6}{7}(2) + c$ $c = \frac{30}{7}$ $y = \frac{6}{7}x + \frac{30}{7}$

(b) Find the length of AB,

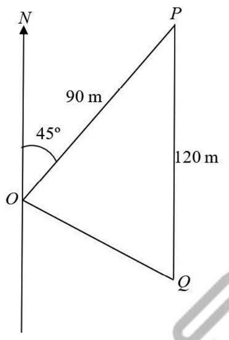
$AB = \sqrt{(6-0)^2 + (2+5)^2} = 9.22$ units

(c) Find the area of triangle ABO.

$\frac{1}{2} \times 5 \times 6 = 15$ units^2

Answer ……….15………… units^2 [1]

17 The diagram shows three points O, P and Q.
The bearing of P from O is 045° and P is due north of Q.
$OP = 90$ m and $PQ = 120$ m.



(a) Find the distance of OQ.

$\angle OPQ = 45°$ (alt. angles, $PQ \parallel ON$)

$OQ^2 = 90^2 + 120^2 - 2(90)(120)\cos 45°$
$= 7226.49$

$OQ = 85.008785$
$= 85.0$

Answer .... m [3]

(b) Calculate the bearing of Q from O.

$\frac{\sin \angle POQ}{120} = \frac{\sin 45°}{85.008785}$

$\sin \angle POQ = \frac{\sin 45°}{85.008785} \times 120$

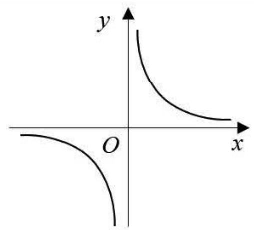
$\angle POQ = 86.529°$

Bearing of Q from O = $45° + 86.529°$
= $131.5°$

Answer ……....………………........... [3]

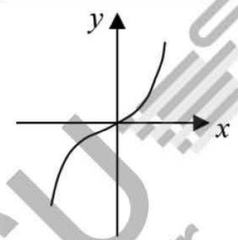
18 Given $y = kx^n$, where $k > 0$. State the value of n for the following graphs.

(a)



Answer $n = -1$ [1]

(b)



Answer $n = 3$ [1]

19 **(a)** Express $-x^2 + 4x + 1$ in the form $-(x+a)^2 + b$.

$-x^2 + 4x + 1$
$= -(x^2 - 4x) + 1$
$= -(x-2)^2 + 4 + 1$
$= -(x-2)^2 + 5$

(b) Hence, solve the equation $-x^2 + 4x + 1 = 0$.

$-(x-2)^2 + 5 = 0$

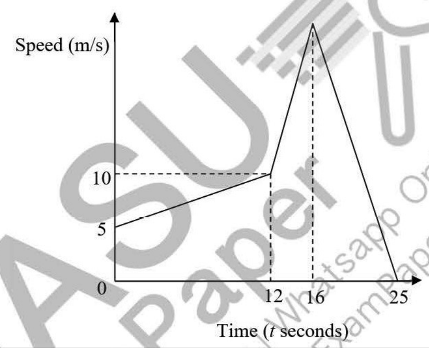
$x - 2 = \pm\sqrt{5}$

$x = \pm\sqrt{5} + 2$

$x = -0.236$ or 4.24

Answer $x =$ …-0.236.…. or …4.24… [2]

20 The diagram shows the speed-time graph of an object moving in a straight line over a period of 25 seconds. The object travelled 90 m from t = 12 to t = 16.



(a) Find the maximum speed of the object.

area under graph = distance travelled

$\frac{1}{2}(4\text{ s})(10+v_{max})=90$

$v_{max}=35\text{ m/s}$

Answer ………………………. m/s [2]

(b) Find the speed of the object at t = 6.

Method 1:	Method 2:
a in first 12 s = $\frac{10\text{ m/s}-5\text{ m/s}}{12\text{ s}}$	$\frac{v-5}{6-0}=\frac{10-5}{12-0}$ $v=7.5$ m/s

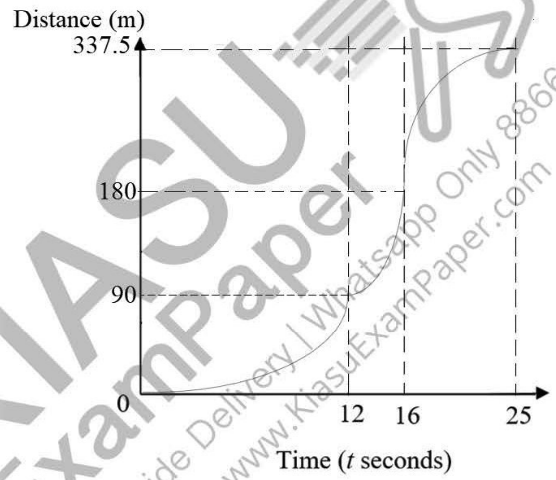
$= \frac{5}{12}\text{ m/s}^2$

speed at $t=6$ $=5\text{ m/s}+(6\text{ s})\left(\frac{5}{12}\text{ m/s}^2\right)$

$=7.5$ m/s

Answer ………………………. m/s [2]

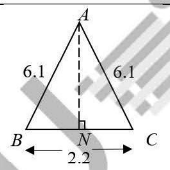
(c) Sketch the distance-time graph for $0 \le t \le 25$.



[3]

S1 for the correct shape for the 3 parts of the journey, without the distances on the vertical axis. Minus 1 mark for 1 wrong value of distance.

21 Triangle ABC has $AB = AC = 6.1$ cm, $BC = 2.2$ cm.



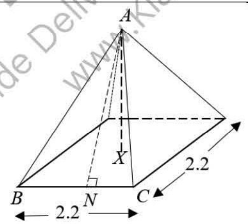
(a) Show that $AN = 6$ cm.

$AC^2 = AN^2 + NC^2$

$AN = \sqrt{6.1^2 - 1.1^2}$ [M1]

$AN = 6$ [A1]

(b) Triangle ABC in (a), is one of the faces of a square-based pyramid.
X is the centre of the base of the pyramid.



(i) Calculate the total surface area of the pyramid.

Total surface area

$= (2.2 \times 2.2) + 4\left(\frac{1}{2} \times 2.2 \times 6\right)$ [M1]

$= 31.24$ cm^2 [A1]

Answer ....31.24.............. cm^2 [2]

(ii) Calculate the volume of the pyramid.

$AX = \sqrt{6^2 - 1.1^2}$
$= 5.8983$

Vol of the pyramid $= \frac{1}{3} \times 2.2^2 \times 5.8983$
$= 9.52$

Answer ....9.52.......... cm^3 [3]