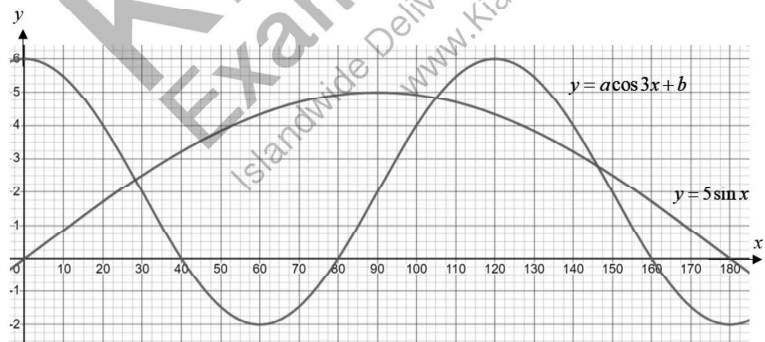
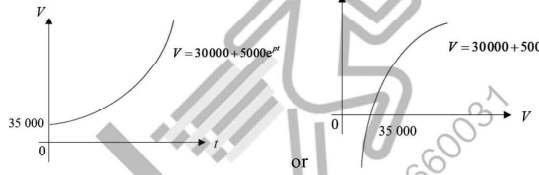


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Answer (BTY 3E AM EOY 2023)

1 $\left(\frac{13}{3}, \frac{4}{3}\right)$ and $(4, 2)$	2(a) $x = -0.631$ (b) $x = 0$ (rejected) or $x = 11$
3(a) $x < 1$ or $x > 5$ (b) Since $k < 0$, $4k^2 > 0$ and $12k < 0$, hence $4k^2 - 12k > 0$. Since discriminant > 0 , the line $y = 2x + 1$ is not a tangent to the curve.	
4(a) $6p - q$ (b) $a = -3$, $b = 4$	5(a) $-5(x-2)(x-4)$ (bi) $\cot X = -\frac{7}{24}$ (bii) $\sec Y = \frac{13}{5}$
6 $a = 2$ and $b = 5$	8(a) $f(x) = (x-1)(x-a)(x-a^2)$ $f(-1) = (-1-1)(-1-a)(-1-a^2) = -8$ $1 + a^2 + a + a^3 = 4$ $a^3 + a^2 + a - 3 = 0$ (shown) (b) there is one solution for $a^3 + a^2 + a - 3 = 0$
9(a) $\frac{2\pi}{3}$ (b) $a = 4$ and $b = 2$	9(d) I do not agree with him. When $y = 5\sin x$ and $y = a\cos 3x + b$ are drawn on the same axes, there are 2 points of intersection, hence 2 solutions for $90^\circ < x < 180^\circ$
9(c) & (d) 	
10(ai) $256 + \frac{1024}{3}x + \frac{1792}{9}x^2 + \frac{1792}{27}x^3 + \dots$	10(bi) $T_{r+1} = \binom{9}{r} (x)^{9-r} \left(\frac{1}{5x^2}\right)^r$ (bii) $9 - 3r$

(aii) $k = 2$	(biii) 1.8
11(a) $V = 35000$ (b) $V = 530000$ (c) 12 years	11(d) 
12(a) $(5, 3)$ and $r = 5$ units (b) $(x-5)^2 + (y-3)^2 = 25$ or $x^2 - 10x + y^2 - 6y + 9 = 0$	12(d) Since the x-coordinate of C is 5 and the radius of the circle is 5 units, C is 5 units from the y-axis, hence the circle touches the y-axis only at one point P. (c) $(0, 0.5)$
13(a) $(0, 12)$ (b) $(8, 11)$ (d) $r = 1.25$ (e) 62.5 units^2	



BEATTY SECONDARY SCHOOL
END-OF-YEAR EXAMINATION 2023
SECONDARY THREE EXPRESS

CANDIDATE NAME

CLASS REGISTER NUMBER

ADDITIONAL MATHEMATICS

4049

9 October 2023

Setter: [Click or tap here to enter text.](#)

2 hours 15 minutes

Candidates answer on the Question Paper
Additional Materials: Nil

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer all the questions.
Given non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [] at the end of each question or part question.

The total of the marks for this paper is 90.

For Examiner's Use

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This document consists of **19** printed pages and **1** blank pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

- 1 The line $2x + y = 10$ intersects the curve $2y^2 = 2xy - 8$.

Find the coordinates of the points of intersection.

[4]

$$2x + y = 10$$

$$y = -2x + 10 \quad \text{----- (1)}$$

$$2y^2 = 2xy - 8 \quad \text{----- (2)}$$

Sub (1) into (2),

$$2(-2x + 10)^2 = 2x(-2x + 10) - 8 \quad \text{[M1 : substitution]}$$

$$8x^2 - 80x + 200 = -4x^2 + 20x - 8$$

$$12x^2 - 100x + 208 = 0$$

$$3x^2 - 25x + 52 = 0$$

$$(3x - 13)(x - 4) = 0 \quad \text{[M1 : correct factorisation or o.e.]}$$

$$x = \frac{13}{3} \text{ or } x = 4$$

sub x into (1), $y = \frac{4}{3} \text{ or } y = 2$

Therefore coordinates of the points of intersection are $\left(\frac{13}{3}, \frac{4}{3}\right)$ and $(4, 2)$.

[A2 : 1 mark for each correct pair of coordinates]

* If students did not leave answers as coordinates, give A1 for both correct x-coordinates or both correct y-coordinates

- 2 (a) Solve $2(3^{2x}) + 3^{x+3} = 14$.

[3]

$$2(3^{2x}) + 3^{x+3} = 14$$

$$2(3^{2x}) + 27(3^x) - 14 = 0$$

$$2(3^x)^2 + 27(3^x) - 14 = 0 \quad \text{[M1 : quadratic equation]}$$

Let $y = 3^x$, $2y^2 + 27y - 14 = 0$

$$(2y - 1)(y + 14) = 0 \quad \text{[M1 : correct factorisation]}$$

$$y = 0.5 \text{ or } y = -14$$

$$3^x = 0.5 \text{ or } 3^x = -14 \text{ (rejected)}$$

$$x = -0.63092 \dots \dots$$

$$= -0.631 \text{ (3 s.f.)} \quad \text{[A1 : with reject } 3^x = -14]$$

- (b) Solve $\log_2(x^2 + 5x) - 4 = \frac{1}{\log_x 2}$.

[4]

$$\log_2(x^2 + 5x) - 4 = \frac{1}{\log_x 2}$$

$$\log_2(x^2 + 5x) - 4 = \log_2 x \quad \text{or} \quad \log_2(x^2 + 5x) - 4 \log_2 2 = \log_2 x \quad \text{[M1 : change of base]}$$

$$\log_2(x^2 + 5x) - \log_2 x = 4 \quad \log_2(x^2 + 5x) - \log_2 2^4 = \log_2 x$$

$$\log_2\left(\frac{x^2 + 5x}{x}\right) = 4 \quad \log_2\left(\frac{x^2 + 5x}{16}\right) = \log_2 x \quad \text{[M1 : apply quotient law or o.e.]}$$

$$\frac{x^2 + 5x}{x} = 16 \quad \frac{x^2 + 5x}{16} = x$$

$$x^2 - 11x = 0 \quad \text{[M1 : quadratic equation]}$$

$$x(x - 11) = 0$$

$$x = 0 \text{ (rejected) or } x = 11 \quad \text{[A1 : with reject } x = 0]$$

[Turn over

3 The equation of a curve is $y = -x^2 + 2kx - k$, where k is a constant.

(a) In the case where $k = 3$, find the set of values of x for which the curve lies completely below the line $y = 2$.

when $k = 3$, $y = -x^2 + 6x - 3$

$$-x^2 + 6x - 3 < 2 \quad [\text{M1 : form correct inequality}]$$

$$-x^2 + 6x - 5 < 0$$

$$x^2 - 6x + 5 > 0$$

$$(x-1)(x-5) > 0 \quad [\text{M1 : factorisation}]$$

$$x < 1 \quad \text{or} \quad x > 5 \quad [\text{A1 with 'or'}]$$

(b) In the case where $k < 0$, explain if the line $y = 2x + 1$ is a tangent to the curve.

$$y = -x^2 + 2kx - k \quad \text{----- (1)}$$

$$y = 2x + 1 \quad \text{----- (2)}$$

Sub (1) into (2),

$$2x + 1 = -x^2 + 2kx - k \quad [\text{M1 : substitution}]$$

$$x^2 + x(2 - 2k) + 1 + k = 0$$

Discriminant $= (2 - 2k)^2 - 4(1)(1 + k) \quad [\sqrt{\text{M1 : finding discriminant}}]$

$$= 4 - 8k + 4k^2 - 4 - 4k$$

$$= 4k^2 - 12k$$

Since $k < 0$, $4k^2 \geq 0$ and $12k < 0$, hence $4k^2 - 12k > 0$.

Since discriminant > 0 , the line $y = 2x + 1$ is not a tangent to the curve. } [A1 : must state $k < 0$]

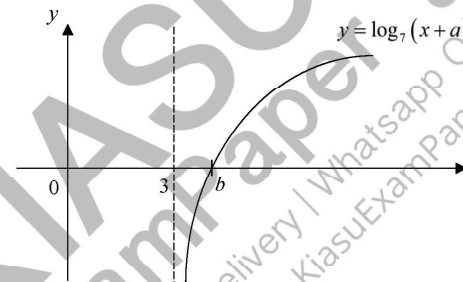
4 (a) Given $\log_a 2 = p$ and $\log_a 5 = q$, express $\log_a 64 + \log_a 0.2$ in terms of p and q . [2]

$$\log_a 64 + \log_a 0.2 = \log_a 2^6 + \log_a 5^{-1} \quad [\text{M1 : either } 2^6 \text{ or } 5^{-1} \text{ seen}]$$

$$= 6\log_a 2 - \log_a 5$$

$$= 6p - q \quad [\text{A1 or A2}]$$

(b) The diagram shows part of the graph of $y = \log_7(x + a)$ and the x -intercept is b , where a and b are constants. Find the values of a and b . [2]



$$x = 3, \quad 3 + a = 0$$

$$a = -3 \quad [\text{B1}]$$

at $(b, 0)$, $a = -3$, $0 = \log_7(b - 3)$

$$b - 3 = 1$$

$$b = 4 \quad [\text{B1}]$$

[Turn over

- 5 (a) Given that (3, 5) is the maximum point of the curve $y = a - 5(x+b)^2$, express the equation of the curve in the form $y = h(x+p)(x+q)$.

$$\begin{aligned} y &= 5 - 5(x-3)^2 && [\text{B1} : a = 5] [\text{B1} : b = -3] \\ &= 5 - 5(x^2 - 6x + 9) \\ &= -5(-1 + x^2 - 6x + 9) \\ &= -5(x^2 - 6x + 8) \\ &= -5(x-2)(x-4) && [\text{A1}] \end{aligned}$$

- (b) Angles X and Y are in the same quadrant such that $\cos X = \frac{7}{25}$ and $\sin X = -\frac{12}{13}$.

Find, **without using a calculator**, the exact value of

- (i) $\cot X$,

$$\begin{aligned} \text{opp} &= -\sqrt{25^2 - 7^2} && [\text{M1} : \cot X = \frac{1}{\tan X} \text{ or see '24'}] \\ &= -24 \\ \therefore \cot X &= -\frac{7}{24} && [\text{A1}] \end{aligned}$$

- (ii) $\sec Y$,

$$\begin{aligned} \text{adj} &= \sqrt{13^2 - (-12)^2} && [\text{M1} : \sec Y = \frac{1}{\cos Y} \text{ or see '5'}] \\ &= 5 \\ \therefore \sec Y &= \frac{13}{5} && [\text{A1}] \end{aligned}$$

- 6 **Without using a calculator**, find the values of the integers a and b such that

$$\frac{a+b\sqrt{5}}{2\sqrt{5}+3} = \frac{2\sqrt{5}+3}{2+\sqrt{5}}. \quad [5]$$

$$\begin{aligned} \frac{a+b\sqrt{5}}{2\sqrt{5}+3} &= \frac{2\sqrt{5}+3}{2+\sqrt{5}} \\ a+b\sqrt{5} &= \frac{(2\sqrt{5}+3)^2}{2+\sqrt{5}} \\ &= \frac{4(5)+12\sqrt{5}+9}{2+\sqrt{5}} && [\text{M1} : \text{expansion of numerator}] \\ &= \frac{29+12\sqrt{5}}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}} && [\sqrt{\text{M1}} : \text{conjugate surds}] \\ &= \frac{58-29\sqrt{5}+24\sqrt{5}-60}{4-5} && [\sqrt{\text{M1}} : \text{expansion of numerator}] \\ &= 2+5\sqrt{5} \\ \therefore a &= 2 \text{ and } b = 5 && [\text{A2}] \end{aligned}$$

OR

$$\begin{aligned} \frac{a+b\sqrt{5}}{2\sqrt{5}+3} &= \frac{2\sqrt{5}+3}{2+\sqrt{5}} \\ (2+\sqrt{5})(a+b\sqrt{5}) &= (2\sqrt{5}+3)^2 \\ 2a+2b\sqrt{5}+a\sqrt{5}+5b &= 20+12\sqrt{5}+9 && [\text{M1} : \text{correct expansion}] \\ 2a+5b+\sqrt{5}(a+2b) &= 29+12\sqrt{5} \\ \text{By comparison, } 2a+5b &= 29 \text{ -----(1)} \\ a+2b &= 12 && [\sqrt{\text{M1}} : \text{form 2 equations}] \\ a &= 12-2b \text{ -----(2)} \\ \text{Sub (2) into (1), } 2(12-2b) + 5b &= 29 && [\sqrt{\text{M1}} : \text{solve S.E.}] \\ b &= 5 \\ \text{Sub } b = 5 \text{ into (2), } a &= 2 \\ \therefore a &= 2 \text{ and } b = 5 && [\text{A2}] \end{aligned}$$

[Turn over

- 7 Express $\frac{8x^2-46}{(x-5)(x+1)}$ in partial fractions.

$$(x-5)(x+1) = x^2 - 4x - 5$$

Using long division,

$$\frac{8x^2-46}{(x-5)(x+1)} = 8 + \frac{32x-6}{(x-5)(x+1)} \quad [\text{M1 : quotient 8, remainder } 32x-6]$$

Let $\frac{32x-6}{(x-5)(x+1)} = \frac{A}{x-5} + \frac{B}{x+1}$ [M1 : see $\frac{A}{x-5} + \frac{B}{x+1}$]

$$32x-6 = A(x+1) + B(x-5)$$

Take $x=5$, $154 = 6A$

$$A = \frac{77}{3} \quad [\text{A1}]$$

Take $x=-1$, $-38 = -6B$

$$B = \frac{19}{3} \quad [\text{A1}]$$

$$\therefore \frac{8x^2-46}{(x-5)(x+1)} = 8 + \frac{77}{3(x-5)} + \frac{19}{3(x+1)} \quad [\text{A1}]$$

[5]

- 8 The cubic polynomial $f(x)$ is such that the coefficient of x^3 is 1 and the roots of $f(x)=0$ are 1, a and a^2 . It is given that $f(x)$ has a remainder of -8 when divided by $x+1$.

- (a) Show that $a^3 + a^2 + a - 3 = 0$.

[3]

$$f(x) = (x-1)(x-a)(x-a^2) \quad [\text{M1}]$$

$$f(-1) = (-1-1)(-1-a)(-1-a^2) = -8 \quad [\sqrt{\text{M1}} : x = -1]$$

$$1 + a^2 + a + a^3 = 4$$

$$a^3 + a^2 + a - 3 = 0 \text{ (shown)} \quad [\text{A1}]$$

- (b) Hence determine the number of solution(s) for $a^3 + a^2 + a - 3 = 0$.

[4]

$$\text{Let } g(a) = a^3 + a^2 + a - 3$$

Try $a=1$, $g(1) = 1^3 + 1^2 + 1 - 3 = 0$ [B1]

$\therefore (a-1)$ is a factor of $g(a)$

$$a^3 + a^2 + a - 3 = 0$$

$$(a-1)(a^2 + 2a + 3) = 0 \quad [\text{M1 : getting the quadratic factor}]$$

$a-1=0$ or discriminant for $(a^2 + 2a + 3=0)$

$$a=1 \quad = 2^2 - 4(1)(3) \quad [\sqrt{\text{M1}} : \text{discriminant}]$$

$$= -8 < 0$$

\therefore there is no real root

\therefore there is one solution for $a^3 + a^2 + a - 3 = 0$. [A1]

[Turn over

9 It is given that $y = a \cos 3x + b$ where a and b are positive integers.

(a) State the period of y in radians. [1]

$$\text{period} = \frac{2\pi}{3} \quad [\text{B1}]$$

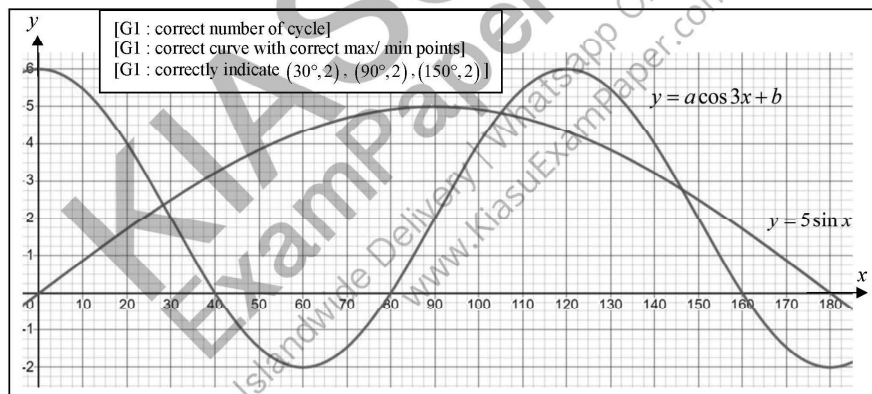
(b) Given that the minimum and maximum values of y are -2 and 6 respectively, find the values of a and b . [2]

$$a = \frac{6 - (-2)}{2} = 4 \quad [\text{B1}]$$

$$b = 6 - 4 = 2 \quad [\text{B1}]$$

Using the values of a and b found in (b),

(c) sketch the graph of $y = a \cos 3x + b$ for $0^\circ \leq x \leq 180^\circ$, [3]



(d) Bryan claimed that there is no obtuse angle x such that $5 \sin x = a \cos 3x + b$.

Do you agree with him? Sketch $y = 5 \sin x$ on the same axes in part (c) and justify your answer. [2]

I do not agree with him.

When $y = 5 \sin x$ and $y = a \cos 3x + b$ are drawn on the same axes, there are 2 points of intersection, hence 2 solutions for $90^\circ < x < 180^\circ$.

[G1 : sketch $y = 5 \sin x$] [√B1 : must mention there is intersection and the range of angles]

[Turn over

10 (a)(i) Write down and simplify the first four terms in the expansion of $\left(2 + \frac{x}{3}\right)^8$ in ascending powers of x . [2]

$$\begin{aligned} \left(2 + \frac{x}{3}\right)^8 &= (2)^8 + \binom{8}{1}(2)^7\left(\frac{x}{3}\right) + \binom{8}{2}(2)^6\left(\frac{x}{3}\right)^2 + \binom{8}{3}(2)^5\left(\frac{x}{3}\right)^3 + \dots \quad [\text{M1}] \\ &= 256 + \frac{1024}{3}x + \frac{1792}{9}x^2 + \frac{1792}{27}x^3 + \dots \quad [\text{A1 : accept mixed numbers}] \end{aligned}$$

(ii) In the expansion of $(3 + kx - x^2)\left(2 + \frac{x}{3}\right)^8$, the coefficient of x^3 is 256, find the value of the constant k . [3]

$$\begin{aligned} (3 + kx - x^2)\left(2 + \frac{x}{3}\right)^8 &= (3 + kx - x^2)\left(256 + \frac{1024}{3}x + \frac{1792}{9}x^2 + \frac{1792}{27}x^3 + \dots\right) \\ \text{Coefficient of } x^3 &= 3\left(\frac{1792}{27}\right) + k\left(\frac{1792}{9}\right) + \frac{1024}{3} \quad [\sqrt{\text{M1}}] \\ \frac{1792}{9}k - \frac{1280}{9} &= 256 \quad [\sqrt{\text{M1 : form equation}}] \\ k &= 2 \quad [\text{A1}] \end{aligned}$$

- (b)(i) Write down the general term in the binomial expansion of $\left(x + \frac{1}{5x^2}\right)^9$. [1]

$$T_{r+1} = \binom{9}{r} (x)^{9-r} \left(\frac{1}{5x^2}\right)^r \quad \text{or} \quad T_{r+1} = \binom{9}{r} \left(\frac{1}{5}\right)^r (x)^{9-3r} \quad [\text{B1}]$$

- (ii) Write down the power of x in this general term. [1]

$$\begin{aligned} (x)^{9-r} \left(\frac{1}{x^2}\right)^r &= (x)^{9-2r} \\ &= (x)^{9-3r} \\ \text{power of } x &= 9-3r \quad [\text{B1 : do not accept } x^{9-3r}] \end{aligned}$$

- (iii) Hence, or otherwise, determine the coefficient of x^6 in the binomial expansion of [2]

$$\left(x + \frac{1}{5x^2}\right)^9$$

$$\begin{aligned} 9-3r &= 6 \quad [\text{M1}] \\ r &= 1 \\ \text{coefficient of } x^6 &= \binom{9}{1} \left(\frac{1}{5}\right)^1 \\ &= 1.8 \quad [\text{A1}] \end{aligned}$$

$$\begin{aligned} \text{OR } \left(x + \frac{1}{5x^2}\right)^9 &= x^9 + \binom{9}{1} (x)^8 \left(\frac{1}{5x^2}\right) + \dots \quad [\text{M1}] \\ \text{coefficient of } x^6 &= \binom{9}{1} \left(\frac{1}{5}\right) \\ &= 1.8 \quad [\text{A1}] \end{aligned}$$

- 11 Andy bought an antique vase. After t years, its value V is given by $V = 30000 + 5000e^{pt}$, where p is a constant. [1]

- (a) Find the value of the vase when Andy bought it.

$$t = 0, \quad V = 35000 \quad [\text{B1}]$$

The value of the vase after 5 years is expected to be \$80 000.

- (b) Calculate the expected value of the vase after 10 years. [3]

$$\begin{aligned} \text{at } t = 5, V = 80000, \quad 80000 &= 30000 + 5000e^{5p} \quad [\text{M1}] \\ e^{5p} &= 10 \\ p &= \frac{\ln 10}{5} \quad [\text{M1}] \\ \text{at } t = 10, \quad V &= 30000 + 5000e^{10\left(\frac{\ln 10}{5}\right)} \\ &= 530000 \quad [\text{A1}] \end{aligned}$$

Accept the following:

$$\begin{aligned} p = 0.4605, \quad V &= 529914.9142 = 529914.91 \quad (2 \text{ d.p.}) \\ p = 0.46051, \quad V &= 529964.9082 = 529964.91 \quad (2 \text{ d.p.}) \\ p = 0.460517, \quad V &= 529999.907 = 529999.91 \quad (2 \text{ d.p.}) \\ p = 0.46052, \quad V &= 530014.9072 = 530014.91 \quad (2 \text{ d.p.}) \end{aligned}$$

[Do NOT accept $p = 0.461$]

[Turn over

- (c) Calculate the number of years Andy should keep the vase for it to be worth at least a million dollars. [2]

at $V = 1000000$,

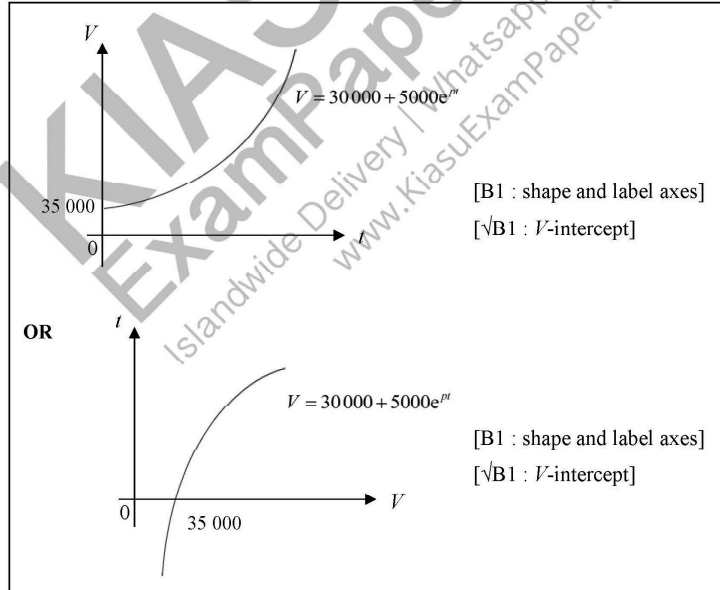
$$1000000 = 30000 + 5000e^{\left(\frac{\ln 10}{5}t\right)} \quad \text{or} \quad 1000000 \leq 30000 + 5000e^{\left(\frac{\ln 10}{5}t\right)} \quad [\sqrt{\text{M1}} - \text{do not accept } <]$$

$$e^{\left(\frac{\ln 10}{5}t\right)} = 194$$

$$t = 11.439$$

Andy should keep the vase for 12 years. [A1 : accept 11.4 and 11.5]

- (d) Sketch the graph of $V = 30000 + 5000e^{0.1t}$ for $t \geq 0$. [2]



- 12 A circle, centre C , has a diameter AB where A is the point $(2, -1)$ and B is the point $(8, 7)$. [3]
- (a) Find the coordinates of C and the radius of the circle.

$$\text{coordinates of } C = \left(\frac{8+2}{2}, \frac{7-1}{2} \right) \\ = (5, 3) \quad [\text{B1}]$$

$$\text{Radius} = \frac{1}{2} \times \sqrt{(8-2)^2 + (7+1)^2} \quad [\text{M1 o.e.}] \\ = 5 \text{ units} \quad [\text{A1}]$$

- (b) Find the equation of the circle. [1]

$$(x-5)^2 + (y-3)^2 = 25 \quad \text{or} \quad x^2 - 10x + y^2 - 6y + 9 = 0 \quad [\text{B1}]$$

[Turn over

- (c) Show that the equation of tangent to the circle at A is $4y + 3x - 2 = 0$. [3]

$$\begin{aligned} \text{gradient of } AB &= \frac{7+1}{8-2} && \text{[M1 : gradient of } AB\text{]} \\ &= \frac{4}{3} \\ \text{gradient of tangent} &= -\frac{3}{4} \\ \text{at } A(2, -1), \quad y+1 &= -\frac{3}{4}(x-2) && \text{or } -1 = -\frac{3}{4}(2) + c \quad \text{[}\sqrt{\text{M1 o.e.}}\text{]} \\ 4y+4 &= -3x+6 && c = 0.5 \\ 4y+3x-2 &= 0 \text{ (shown)} && y = -\frac{3}{4}x + \frac{1}{2} \quad \text{[A1]} \\ &&& 4y = -3x + 2 \\ &&& 4y + 3x - 2 = 0 \text{ (shown)} \end{aligned}$$

- (d) Explain why the circle touches the y -axis only at one point P . [2]

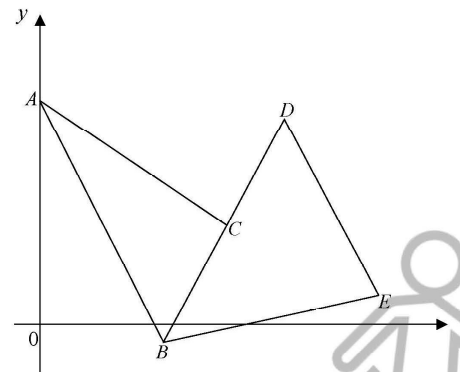
Since the x -coordinate of C is 5 and the radius of the circle is 5 units, [B1]
 C is 5 units from the y -axis, hence the circle touches the y -axis only at one point P . [B1]

OR on y -axis, $x = 0$, $(0-5)^2 + (y-3)^2 = 25$ [√M1]
Conclusion 1 : When $x = 0$, there is only one real root for y , $y = 3$, hence the circle touches the y -axis only at one point P . **OR**
Conclusion 2 : Since the discriminant for $y^2 - 6y + 9 = 0$ is $(-6)^2 - 4(1)(9) = 0$, the circle touches the y -axis only at one point P . [A1]

- (e) Find the coordinates of the point at which the tangents to the circle at A and P intersect. [2]

$$\begin{aligned} x = 0, \quad 4y + 3(0) - 2 &= 0 && \text{[M1]} \\ y &= 0.5 \\ \text{coordinates of the point of intersection} &= (0, 0.5) && \text{[A1]} \end{aligned}$$

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The diagram shows a pentagon $ABEDC$ where A lies on the y -axis, and coordinates of B and C are $(4, -1)$ and $(6, 5)$ respectively. The equation of the line AB is $13x + 4y = 48$.

- (a) Find the coordinates of A . [1]

$$\begin{aligned} x = 0, \quad 13(0) + 4y &= 48 \\ y &= 12 \\ \text{coordinates of } A &= (0, 12) && \text{[B1]} \end{aligned}$$

The line BC is extended to the point D such that C is the midpoint of BD .

- (b) Find the coordinates of D . [2]

$$\begin{aligned} \text{Let coordinates of } D &= (p, q) \\ \left(\frac{p+4}{2}, \frac{q-1}{2} \right) &= (6, 5) && \text{[M1 : o.e. Do NOT accept } m_{BC} = m_{BD}\text{]} \\ p = 8 \quad \text{and} \quad q &= 11 \\ \text{coordinates of } D &= (8, 11) && \text{[A1]} \end{aligned}$$

[Turn over

- (c) Determine, with working, if angle ADB is a right angle. [2]

$$\begin{aligned} & \text{gradient of } AD \times \text{gradient of } DB \\ &= \frac{12-11}{0-8} \times \frac{11+1}{8-4} \quad [\sqrt{M1} : \text{o.e.}] \\ &= -\frac{3}{8} \neq -1 \\ & \therefore \text{angle } ADB \text{ is not a right angle } \quad \left. \vphantom{\frac{12-11}{0-8}} \right\} [A1] \end{aligned}$$

DE is parallel to AB and the coordinates of E is $(11, r)$.

- (d) Find the value of r . [2]

$$\begin{aligned} & \text{gradient of } AB = \text{gradient of } DE \\ & \frac{-13}{4} = \frac{11-r}{8-11} \quad [\sqrt{M1}] \\ & r = 1.25 \quad [A1] \end{aligned}$$

- (e) Find the area of the figure $ABEDC$. [2]

$$\begin{aligned} \text{Area} &= \frac{1}{2} \begin{vmatrix} 0 & 4 & 11 & 8 & 6 & 0 \\ 12 & -1 & 1.25 & -11 & 5 & 12 \end{vmatrix} \quad [\sqrt{M1} : \text{o.e.}] \\ &= -\frac{1}{2}(238-113) \\ &= 62.5 \text{ units}^2 \quad [A1] \end{aligned}$$

[Turn over