NAME: __MARK SCHEME______________________ ( ) MARKS: _______ /90

CLASS: _______

ADDITIONAL MATHEMATICS

MONDAY **3 OCTOBER 2022** **2 HOUR 15 MINUTES**

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JUYING SECONDARY SCHOOL

END-OF-YEAR EXAMINATION

SECONDARY THREE EXPRESS

Instructions to students:

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

Read these notes carefully.

1. Answer **all** questions.
2. Write in dark blue or black pen in the spaces provided.
3. You may use an HB pencil for any diagrams or graphs.
4. Omission of essential working will result in loss of marks.
5. The use of an approved calculator is expected, where appropriate.
6. If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
7. The number of marks is given in brackets [ ] at the end of each question or part question.
8. The total number of marks for this paper is 90.

This question paper consists of 19 printed pages, including this page.

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Vetter: Mdm Norhafiani A Majid

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

1. Solve $\sqrt{7-6x}+2=x$. [4]

$\sqrt{7-6x}+2=x$

$\sqrt{7-6x}=x-2$

$7-6x=(x-2)^2$ [M1]

$7-6x=x^2-4x+4$

$x^2+6x-4x+4-7=0$

$x^2+2x-3=0$ [M1]

$(x-1)(x+3)=0$ [M1]

$x-1=0$ or $x+3=0$

$x=1$ $\quad x=-3$ [A1 deduct if not even 1 ans is rejected]

Both x values do not satisfy the equations, there are no solution.

2. The curved surface area of a cylinder is $(6\sqrt{3}+16)\pi$ cm². The radius of the base of the cylinder is $(1+\sqrt{3})$ cm. Find, without using a calculator, the height of the cylinder in the form $a+b\sqrt{3}$, where a and b are rational numbers. Show your steps clearly. [4]

$2\pi rh=(6\sqrt{3}+16)\pi$

$2\pi(1+\sqrt{3})h=(6\sqrt{3}+16)\pi$ [M1]

$h=\dfrac{3\sqrt{3}+8}{1+\sqrt{3}}$

$h=\dfrac{3\sqrt{3}+8}{1+\sqrt{3}}\times\dfrac{1-\sqrt{3}}{1-\sqrt{3}}$ [M1]

$h=\dfrac{(3\sqrt{3}+8)(1-\sqrt{3})}{1-3}$

$h=\dfrac{3\sqrt{3}-9+8-8\sqrt{3}}{1-3}$ [M1]

$h=\dfrac{-1-5\sqrt{3}}{-2}$

$h=\dfrac{1}{2}+\dfrac{5}{2}\sqrt{3}$ [A1]

3. Solve the simultaneous equations [5]

$$3^y=\tfrac{1}{3}(3^{x-7}),$$

$$\log_9(14-2y)-\log_9 x=0.5.$$

$3^y=\dfrac{1}{3}(3^{x-7})$

$3^y=3^{x-8}$

$y=x-8$ - eqn 1 [M1]

$\log_9(14-2y)-\log_9 x=0.5$

$\log_9\frac{(14-2y)}{x}=0.5\log_9 9$ [M1]

$\log_9\frac{(14-2y)}{x}=\log_9 9^{0.5}$

$\dfrac{(14-2y)}{x}=3$

$14-2y=3x$ - eqn 2 [M1]

Sub (1) into (2),

$14-2(x-8)=3x$

$14-2x+16=3x$

$30=5x$

$x=6$ [M1]

Sub $x=6$ into (1),

$y=-2$ [A1]

4. (a) Express $y = -3x^2 + 6x + 24$ in the form of $y = a(x+b)^2 + c$, where a, b and c are constants. [2]

$y = -3x^2 + 6x + 24$

$= -3(x^2 - 2x) + 24$

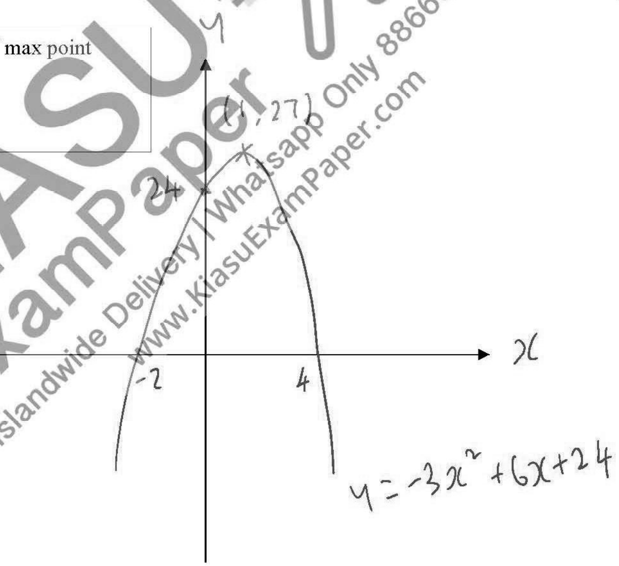
$= -3(x^2 - 2x + 1 - 1) + 24$

$= -3[(x-1)^2 - 1] + 24$

$= -3(x-1)^2 + 27$ [B1/B1]

(b) Sketch the graph of $y = -3x^2 + 6x + 24$. Label the turning point and y-intercept. [2]

B1 max curve shape / max point
B1 y intercept



(c) Hence, explain why y cannot exceed 27. [1]

Since $(x-2)^2 \geq 0$ for all real values of x, OR the maximum point is (1,27), the maximum value of y =27 and thus y cannot exceed 27 [B1]

5. The cubic polynomial $f(x) = 3x^3 + hx^2 - 14x + k$ leaves a remainder of 40 when divided by $x - 3$ and it is exactly divisible by $x + 2$.

(a) Show that $h = 1$ and $k = -8$ [4]

$f(3) = 40$

$3(3)^3 + h(3^2) - 14(3) + k = 40$

$81 + 9h - 42 + k = 40$

$9h + k = 1$ (eqn 1) [M1]

$f(-2) = 0$

$3(-2)^3 + h(-2)^2 - 14(-2) + k = 0$

$-24 + 4h + 28 + k = 0$

$k = -4 - 4h$ (eqn 2) [M1]

Sub (2) into (1), $9h - 4 - 4h = 1$

$5h = 5$

$h = 1$ [A1]

sub $h = 1$ into (2), $k = -8$ [A1]

(b) Hence, solve the cubic equation $f(x) = 0$. [4]

$3x^3 + x^2 - 14x - 8 = 0$

$f(-2) = 0$

$(x + 2)$ is a factor

$(x + 2)(3x^2 - 5x - 4) = 0$ [M1 for quadratic factor]

$x + 2 = 0$ or $3x^2 - 5x - 4 = 0$

$x = -2 \qquad x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(-4)}}{2(3)}$ [M1]

$x = -2, \qquad x = 2.26, \quad x = -0.591$ (3sf) [A1/A1]

6. Find the term independent of x in the expansion of $\left(x^3 - \frac{3}{x}\right)^8$. [3]

$T_{r+1} = \binom{8}{r}(x^3)^{8-r}\left(-\frac{3}{x}\right)^r$ [M1]

$T_{r+1} = \binom{8}{r}x^{24-3r}(-3)^r\left(\frac{1}{x}\right)^r$

$T_{r+1} = \binom{8}{r}(-3)^r x^{24-4r}$

$24 - 4r = 0$

$r = 6$ [M1]

Term independent of $x = 20412$ [A1]

7. (a) Write down and simplify the first three terms in the expansion of $(2 - 3x)^6$. [2]

$= 2^6 + \binom{6}{1}2^5(-3x) + \binom{6}{2}2^4(-3x)^2 + \cdots$ [M1]

$= 64 - 576x + 2160x^2 + \cdots$ [M1]

(b) Hence, find the value of p if the coefficient of the $\frac{1}{x}$ term in the expansion $\left(\frac{1}{x} - \frac{p}{x^2}\right)(2 - 3x)^6$ is 1216. [3]

$\left(\frac{1}{x} - \frac{p}{x^2}\right)(64 - 576x + 2160x^2 + \cdots)$

$= \frac{64}{x} - \frac{p}{x^2}(-576x) + \cdots$

$= \frac{64}{x} + \frac{576p}{x} + \cdots$

Coefficient of $\frac{1}{x} = 1216$

$64 + 576p = 1216$ [M1, M1 – for each coefficeient of $\frac{1}{x}$]

$576p = 1152$

$p = 2$ [A1]

8. The population of a species is measured daily. The results are tabulated in the table below.

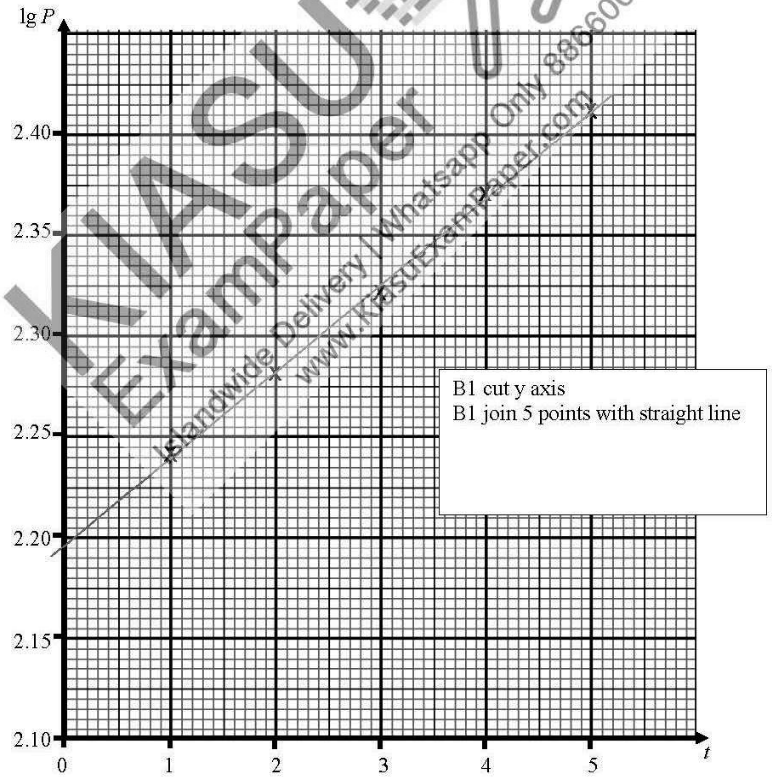
Number of days, t	1	2	3	4	5
Population, P	174	192	211	232	255
lg P	2.24	2.28	2.32	2.37	2.41

(a) It is known that t and P are related by the equation $P = ab^t$, where a and b are constants.

Express this equation in a form suitable for plotting a straight line graph of lg P against t. [2]

$P = ab^t$
$lgP = lgab^t$ [M1]
$lgP = lga + lgb^t$
$lgP = tlgb + lga$ [A1]

(b) On the grid below plot lg$P$ against t and draw a straight line graph. [2]



(c) Explain what the value of a represents. [1]

It is the initial population of the species.
It is the population when $t = 0$. [B1]
It is the population at the beginning.

(d) (i) Use your graph to estimate the value of a. [2]

y-intercept = 2.195 [M1]
$lga = 2.195$
$lga = 2.195lg10$
$a = 10^{2.195}$
$a = 156.67$
$a = 157\ (3sf)$ [A1]

(ii) Use your graph to estimate the value of b. [2]

gradient = $\frac{2.41-2.28}{5-2}$ [M1]
$= \frac{13}{300}$

$lgb = \frac{13}{300}lg10$
$b = 10^{\frac{13}{300}}$ [A1]
$b = 1.10\ (3sf)$

9. (a) State the values between which the principal values of $\tan^{-1}x$ must lie. [1]

$-90° < tan^{-1}x < 90°$ [B1] accept $\frac{\pi}{2}$

(b) Express the principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ in radians as a multiple of π. [1]

$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$ [B1]

10. It is given that $y_1 = 3\cos bx + c$ and $y_2 = -1 + 2\sin 3x$.

(a) Find the value of c, given that the maximum value of y_1 is 4. [1]

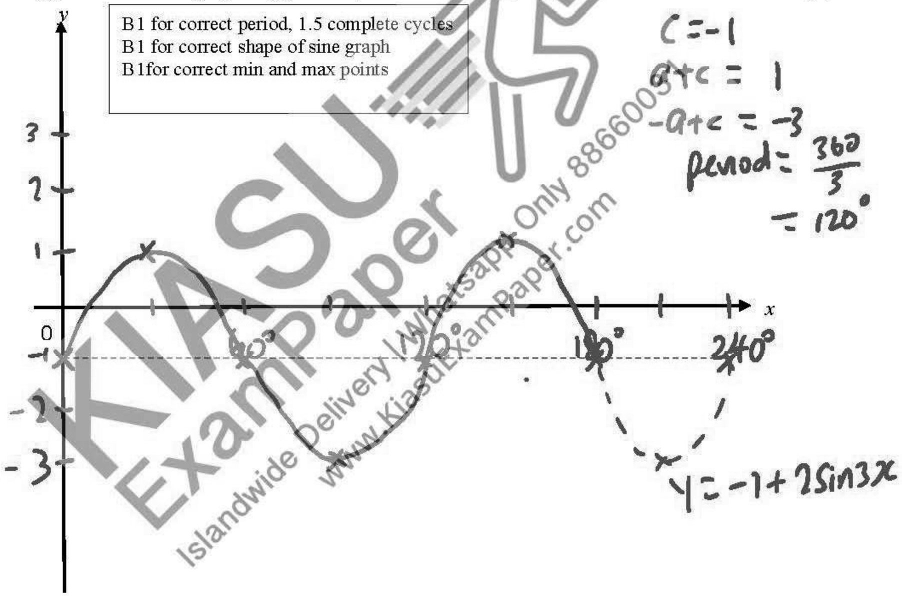
$a + c = 4$
$c = 1$ [B1]

(b) Find the value of b, given that the period of y is 180°. [1]

b = 2 [B1]

(c) Sketch the graph of $y_2 = -1 + 2\sin 3x$ for $0 \leq x \leq 180^0$. [3]

B1 for correct period, 1.5 complete cycles
B1 for correct shape of sine graph
B1for correct min and max points



(d) Explain how the graphical method can be used to find the solutions of $2\sin 3x - 3\cos bx = c + 1$ for $0 \leq x \leq 360^0$. [1]

Sketch the graph of y=3 cos $bx + c$ on the graph in (b) and find the points of intersections of the 2 curves. [B1]

11. Without using a calculator, find the exact value of $\frac{\sin 45°}{\cos 60°}$.
Show all workings clearly. [1]

$$\frac{\sin 45°}{\cos 60°} = \frac{\sqrt{2}}{2} \div \frac{1}{2}$$
$$= \sqrt{2}$$ [B1]

12. (a) Prove that $\frac{1+cos2x}{sin2x} = cotx$. [2]

$$\frac{1+cos2x}{sin2x} = \frac{1+2\cos^2 x-1}{2sinxcosx}$$ [M1 for either]
$$= \frac{2\cos^2 x}{2sinxcosx}$$
$$= \cot x$$ [A1]

(b) Hence, solve $\frac{1+cos4x}{sin4x} = -\frac{1}{3}$ for $0 \leq x \leq 2\pi$. [3]

$\cot 2x = -\frac{1}{3}$
$tan2x = -3$ [M1]

$\propto = \tan^{-1}(3)$
$\propto = 1.2490$

$2\theta = 1.8925, 5.0341, 8.1756, 11.317$ [A1, A1]

$\theta = 0.946,\ 2.52,\ 4.09,\ 5.66$

13. Express $\frac{x^2+4x-2}{x^2-x}$ in partial fractions. [5]

$\frac{x^2+4x-2}{x^2-x} = 1 + \frac{5x-2}{x^2-x}$

$= 1 + \frac{5x-2}{x(x-1)}$ M1 by long division

$\frac{5x-2}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1}$

$5x-2 = A(x-1)+Bx$ [M1,M1]

Sub $x = 1$

$B = 3$ [M1]

Sub $x = 0$

$-2 = -A$

$A = 2$ [M1]

$\frac{x^2+4x-2}{x^2-x} = 1 + \frac{2}{x} + \frac{3}{x-1}$

14. A circle has the equation $x^2 + y^2 - 2x - 6y = 22$ with centre C.

(a) Find the coordinates of the centre and radius of the circle. [2]

Centre $= \left(\frac{-2}{-2}, \frac{-6}{-2}\right)$ [B1]

$= (1, 3)$

Radius $= \sqrt{1+9-(-22)}$

$= \sqrt{32}$

$= 5.66$ units [B1]

(b) The equation of the circle represents the area of Wi-Fi signal coverage where the centre of the circle is the location of the Wi-Fi modem.

Explain if an individual standing at a point $X(4,2)$ is able to receive Wi-Fi signal on his electronic device. Show your working. [2]

Length of CX

$= \sqrt{(4-1)^2+(2-3)^2}$

$= \sqrt{10}$

$= 3.16$ M1

Since the length of CX is less than the radius of the circle, the person is standing inside the circle and is able to receive Wif-Fi signal. A1

(c) A point $P(5, q)$ lies on the circle where $q < 0$. A line AB passing through point P is perpendicular to the line CP. Find the equation of AB. [3]

$5^2 + y^2 - 2(5) - 6y = 22$
$y^2 - 6y - 7 = 0$
$(y - 7)(y + 1) = 0$
$y = 7\ (rej)\ or\ y = -1$ [M1]

$P(5, -1)$

Gradient of $CP = \frac{3-(-1)}{1-5}$
$= -1$ [M1]
Gradient of $AB = 1$

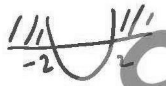
Equation of AB:
$y - (-1) = 1(x - 5)$
$y + 1 = x - 5$
$y = x - 6$ [A1]

15. Find the range of k where the line $y = x + k$ and the curve $x^2 + y^2 = 2$ do no intersect. [4]

$x^2 + y^2 = 2$ - eqn (1)
$y = x + k$ - eqn (2)

Sub (2) into (1),
$x^2 + (x + k)^2 = 2$
$x^2 + x^2 + 2kx + k^2 = 2$
$2x^2 + 2kx + k^2 - 2 = 0$ [M1]

$b^2 - 4ac < 0$
$(2k)^2 - 4(2)(k^2 - 2) < 0$ [M1]
$4k^2 - 8k^2 + 16 < 0$
$-4k^2 + 16 < 0$
$-k^2 + 4 < 0$
$k^2 - 4 > 0$ [M1]
$(k + 2)(k - 2) > 0$

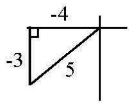


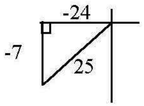
$k < -2\ or\ k > 2$ [A1]

16. Given that $\sin A = -\frac{3}{5}$ and $\cos B = -\frac{24}{25}$ where A and B are in the same quadrant.

Calculate the value of

(a) $\sec A$, [2]





$cosA = -\frac{4}{5}$

$\sec A = \frac{1}{cosA}$ M1

$= -\frac{5}{4}$ A1

(b) $\sin(A+B)$, [2]

$\sin(A+B) = sinAcosB + cosAsinB$

$= \left(-\frac{3}{5}\right)\left(-\frac{24}{25}\right) + \left(-\frac{4}{5}\right)\left(-\frac{7}{25}\right)$ M1

$= \frac{4}{5}$ A1

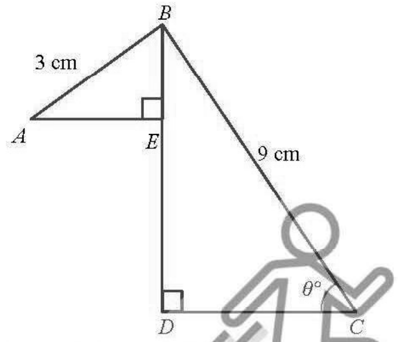
(c) $\tan 2A$. [2]

$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$

$= \frac{2\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)^2}$ M1

$= \frac{24}{7}$ A1

17. The figure shows two right angled triangles ABE and BCD. Line AB is perpendicular to the line BC. It is given that $\angle BCD = \theta°$, AB = 3 cm and BC = 9 cm.



(a) Show that the horizontal distance, L, between point A and C, in terms of θ is given by

$$L = 3sin\theta + 9cos\theta.$$ [2]

$sin\theta = \frac{AE}{3}$

$AE = 3sin\theta$ [B1]

$cos\theta = \frac{CD}{9}$

$CD = 9cos\theta$ [B1]

$L = 3sin\theta + 9cos\theta$ (shown)

(b) Express $3sin\theta + 9cos\theta$ in the form $Rcos(\theta - \propto)$ where $R > 0$ and $\propto$ is acute. [3]

$3sin\theta + 9cos\theta = Rcos(x - \propto)$

$9cos\theta + 3sin\theta = Rcos(x - \propto)$

$R = \sqrt{9^2 + 3^2}$

$R = \sqrt{90}$ [B1]

$\propto = \tan^{-1}\left(\frac{3}{9}\right)$ [B1]

$\propto = 18.435°$

$9cos\theta + 3sin\theta = \sqrt{90}cos(x - 18.4°)$ [A1]

(c) State the maximum value of L and the corresponding value of θ. [3]

Max value = $\sqrt{90}$ [B1]

$\sqrt{90}cos(x - 18.4°) = \sqrt{90}$
$cos(x - 18.4°) = 1$

$\propto = \cos^{-1}(1)$
$\propto = 0°$ [M1]

$x - 18.4° = 0°$
$x = 18.4°$ [A1]

(d) Find the value(s) of θ when $L = \sqrt{55}$ cm. [3]

$\sqrt{90}cos(x - 18.4°) = \sqrt{55}$
$cos(x - 18.4°) = \frac{\sqrt{55}}{\sqrt{90}}$ [M1]

$\propto = \cos^{-1}\left(\frac{\sqrt{55}}{\sqrt{90}}\right)$
$\propto = 38.580°$ [M1]

$x - 18.4° = 38.580°,\ 360° - 38.580°$
$x = 56.98°,\ 339.855°$
$x = 57.0°,\ 339.9°\ (rej)$ [A1]