S4A TOPICAL INTENSIVE REVISION WEEK 2

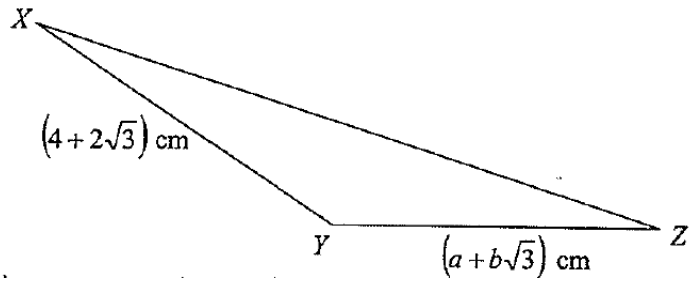
Total Marks: 30

Topic: Surds

1. Solve the equation $x + 1 + \sqrt{5 - x} = 0$. [3]

2. (i) Express $\frac{30+18\sqrt{3}}{2+\sqrt{3}}$ in the form $r + s\sqrt{3}$, where r and s are integers. [3]

   (ii) The diagram shows a triangle XYZ.



   XY is $(4 + 2\sqrt{3})$ cm and YZ is $(a + b\sqrt{3})$ cm, where a and b are integers .

   The included angle XYZ is 150°. Given that the area of the triangle is $(15 + 9\sqrt{3})$ cm^2, find the value of a and of b. [5]

3. (a) Given that $3^{x+2} \times 5^{x-1} = 9^x \times 5^{2x}$, evaluate 15^x without using a calculator. [3]

   (b) Given that n is a positive integer, explain why the largest prime factor of $2(5^{n+1}) - 4(5^n) + 5^{n+2}$ is 31. [3]

4. (a) Solve $9^{x+1} - 3^{x+2} = 2(3^{x+1} - 2)$. [4]

   (b) Given that $\log_3(2x - 1) = \log_9(2x - 1) + 1$, find the value of x. [3]

   (c) Given that $\log_a xy = m$ and $\log_a \frac{x^3}{y^2} = n$, find $5 \log_a (\frac{x}{y})$ in terms of m and n. [5]

5. Solve the equation $\log_5 x = 3 - 2 \log_x 5$. [4]

6. Solve the equation $\frac{1}{\log_9 3} + \log_3(2x + 6) = \log_{\sqrt{3}}(1 - x) + 1$. [5]

7. A certain radioactive substance is known to decay with time such that its mass, M g after t hours is given to be $M = M_0 e^{-kt}$, where k is a positive constant. A block of this substance having a mass of 100g originally is observed. After 40 hours, its mass has decreased to 90g. Find

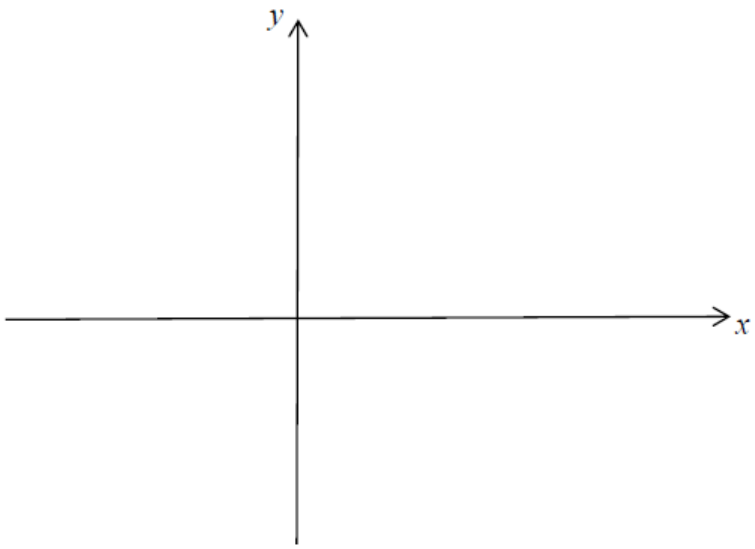
   (i) the value of k. [3]

   (ii) the time lapse to the nearest hour when the block first decays to half of its original mass. [3]

8. The graph of $y = \log_p x$ passes through the points $(27, 3)$ and $(q, -2)$.

   (a) Find the value of p and of q. [2]

   (b) Sketch on the same diagram, the graphs of $y = \log_p x$ and $y = 3^{-x}$. [3]



   (c) State the number of solutions of for $\log_p x = 3^{-x}$. [1]

Answer Key

1(i).	$x = -4$
2(i).	$6 + 6\sqrt{3}$
2(ii).	$a = 6, b = 6$
3a.	$15^x = \frac{9}{5}$
3b.	$5^n(31)$
4a.	$x = -1$ or $x = 0.262$
4b.	$x = 5$
4c.	$2n - m$
5.	$x = 25$ *or* 5
6.	$x = -1.74$
7i.	$k = 0.00263$
7ii.	$t = 264$
8a.	$p = 3, q = \frac{1}{9}$
8c.	1 solution