Total Marks: 65

Topic: Coordinate Geometry & Circles

1. The diagram, which is not drawn to scale, shows a trapezium PQRS in which PQ is parallel to SR. The point P lies on the y-axis and the point Q is (7,9). The point T(6,7) lies on PR such that QT is perpendicular to PR and $QS = 3\,QT$.

    (i) Find the equation PR. [2]

    (ii) Hence, write down the coordinates of P. [1]

    (iii) Find the coordinates of V if PQRV is a kite. [2]

    (iv) Show that the coordinates of S are (4,3). [2]

   (v) Find the coordinates of R if the area of PQRS is 67.5 square units. [3]
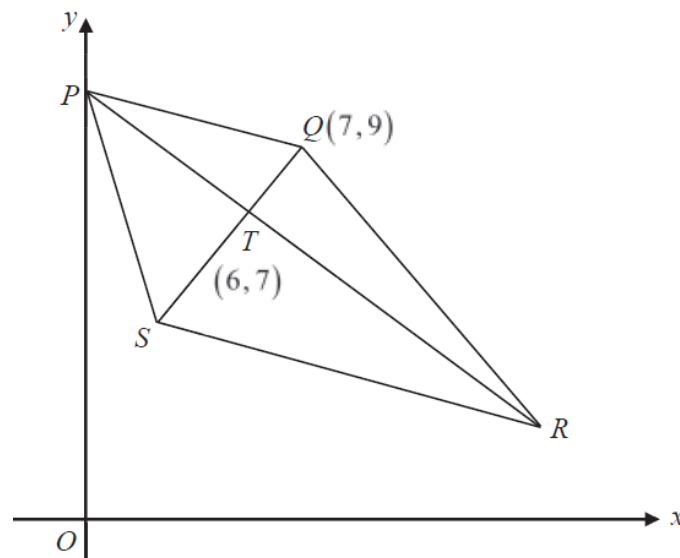
2. The diagram shows a parallelogram OABC, where O is the origin and A is the point (-6,3). The equation of OA, OC and BC are $x + 2y$, $y = \frac{1}{3}x$ and $x + 2y = 20$ respectively.

(a) Find the coordinates of C and of B. [4]

The perpendicular from A to OC meets CO produced at the point F.

(b) Find the coordinates of F. [3]

(c) Calculate the exact length of AF and hence find the area of the parallelogram OABC. [3]
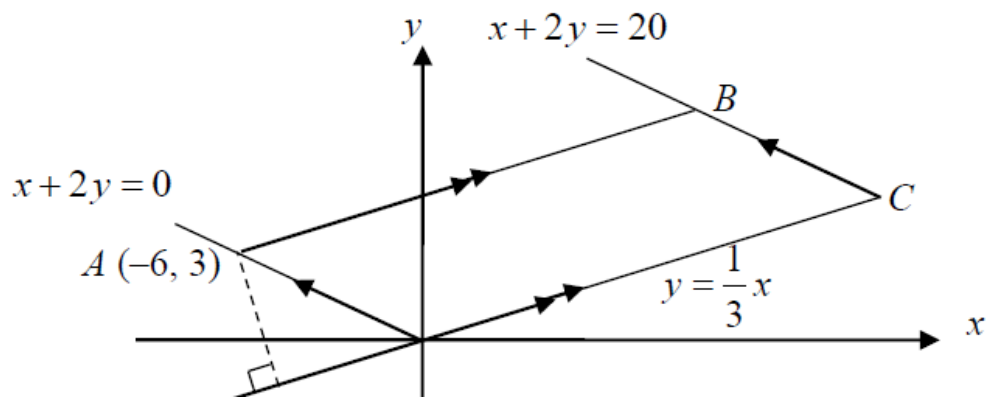
3. The points P (-2,-6), Q(2,-8) and R(6,0) lie on a circle.

(i)   Show that PR is a diameter of the circle. [3]

(ii) Find the equation of the circle. [3]

(iii) Find the possible equations of tangent to the circle which is parallel to the x −axis. [2]

(iv) Find the equation of the perpendicular bisector of QR and show that it passes through the centre of the circle. [3]

4. The equation of a circle C_1 is $x^2 + y^2 - 14x + 2y + 46 = 0$

   (i) Find the radius of circle C_1 and the coordinates of its centre. [4]

   Circle C_1 is reflected about the line $x = 4$ to form another circle C_2 with centre M.

   (ii) Find the coordinates of M. [1]

   (iii) Find equation of circle C_2. [1]

   A line, l passing through M, forms an angle θ with the positive x-axis such that $\tan \theta = 1$

   (iv) Find equation of line l. [1]

   The tangent to circle C_1 at the point (8.2, 0.6) meets line l at Q.

   (v) Find the coordinates of Q. [5]

5. A circle, C_1, passes through the points $A(0, 9)$ and $B(-3, 0)$. Its centre, C, lies on the line $y = x + 8$.

   (a)(i) Show that the centre of the circle, C_1 is (-3,5) [4]

   (ii) Hence find the equation the circle C_1. [2]

   (b) A second circle, C_2, with the equation $x^2 + y^2 + ax + by - 2 = 0$, has the same centre, C as circle C_1.

   (i) Find the value of a and of b. [2]

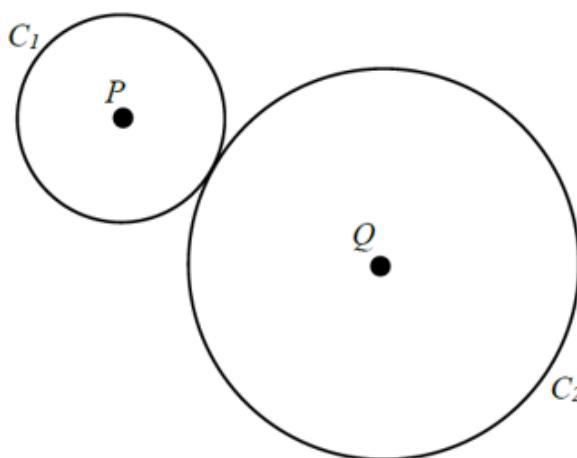   (ii) Explain why the circle C_2 lies outside of the circle C_1. [3]

6. A circle C_1 is given by the equation $x^2 + y^2 + 10x - 6y + 9 = 0$. Another circle C_2 with the centre $Q(7, -2)$ touches the circle C_1 externally as shown in the diagram below.

   (i) Find the coordinates of the centre P and of the radius of circle C_1.   [3]

   (ii) Show that the equation of circle C_2 is $(x - 7)^2 + (y + 2)^2 = 64$.   [2]

   It is given that the equation of the tangent to the circle C_1 at the point R is $3y + 4x = k$.

   (iii) Given that the point R lies to the left of centre P, find the value of the constant k.   [6]

MATRIX MATH

Answer Key

1(i).	$y = -\dfrac{1}{2}x + 10$
1(ii).	$P(0,10)$
1(iii).	$V(5,5)$
1(v).	$R(18,1)$
2(a).	$C(12,4), B(6,7)$
2(b).	$F(-4.5, -1.5)$
2(c).	$\dfrac{3}{2}\sqrt{10}, 60$
3(ii).	$(x-2)^2 + (y+3)^2 = 25$
3(iii).	$y = 2, y = -8$
3(iv).	$y = -\dfrac{1}{2}x - 2$
4(i).	$(7, -1), 2$ units
4(ii).	$M(1, -1)$
4(iii).	$x^2 + y^2 - 2x + 2y - 2 = 0$
4(iv).	$y = x - 2$
4(v).	$Q(5,3)$
5(aii).	$(x+3)^2 + (y-5)^2 = 25$
5(bi).	$a = 6, b = -10$
5(ii).	Since the radius of C_2 is longer that of C_1, circle C_2 lies outside of the circle C_1.
6(i).	$P(-5,3), r = 5$
6(iii).	$k = -36$