S3A TOPICAL INTENSIVE REVISION WEEK 4

Total Marks: 30

Topic: Exponential

1 (a) Solve the equation $3^{2x+2} - 10\left(3^x\right) + 1 = 0$. [4]

(b) Without using a calculator, evaluate 15^x given that [3]
$3^{2x-1} \times 5^{4-x} = 3^{x+3} \times 5^{3-2x}$.

2 Solve

(a) $9^x + 2\left(3^x\right) = 3^{x+2} - 12$, [4]

(b) $e^x = 5 + 6e^{-x}$. [3]

3 Given that $\dfrac{9^{n+2} - 3^{2n+2}}{2^5} = 2^a 3^b$, where a and b are integers,

(i) find the value of a and express b in terms of n, [5]

(ii) hence, or otherwise, solve the equation $\dfrac{9^{n+2} - 3^{2n+2}}{2^5} = \dfrac{1}{4}$. [2]

4 Prove that $2^x + \dfrac{1}{2}\left(2^{x+4}\right) - 2^{x+2}$, where x is a positive integer, is exactly [3]
divisible by 5.

5 The quantity, N, of a particle decaying is given by $N = 3500 + 2000e^{-0.04t}$ where t is the time in years after the particle starts decaying.

(i) Find the quantity of the particle at which the particle has not started decaying. [1]

(ii) Find the quantity of the particle when $t = 14$. [1]

(iii) Express t in terms of N. [3]

(iv) Explain why the quantity of the particle can never reach 3500. [1]

Answer Key

1(a)	$x = -2, 0$
1(b)	$\frac{81}{5}$
2(a)	$x = 1, 1.26$
2(b)	1.79
3(i)	$a = -2,\ b = 2n + 2$
3(ii)	$n = -1$
4	$\left(2^x\right)(5)$ is a multiple of 5, hence divisible by 5.
5(i)	5500
5(ii)	4640
5(iii)	$t = -25\ln\left(\frac{N - 3500}{2000}\right)$
5(iv)	As $t \to \infty$, $2000e^{-0.04t} \to 0$, $3500 + 2000e^{-0.04t} \to 3500$