Topic: Binomial Theorem & Linear Law

Total marks: 43

1. (a) Write down the first three terms in the expansion of $(2-\frac{x}{4})^n$, where n is a positive integer greater than 2, in ascending powers of x. [4]

The first two non-zero terms in the expansion of $(2+x)(2-\frac{x}{4})^n$ in ascending powers of x are $a+bx^2$, where a and b are constants.

(b) Find the value of n. [2]

(c) Hence, find the value of a and of b. [2]

2. (a) The expression of $(1+px)^n$ where $n>0$ by the binomial theorem is $1+14x+21p^2x^2+kx^3+\cdots$, find the value of p and of k. [5]

(b) Evaluate the coefficient of x^5 in the expansion of $(x^2-\frac{2}{x})^7$. [4]

3. (i) Explain why there is no constant term in the expansion of $(x^3-\frac{1}{x^2})^8$. [2]

(ii) Show that the coefficient of x^{-6} in the expansion of $\left(x^3-\frac{1}{x^2}\right)^8(1+x^5)$ is 20. [4]

4. The table shows experimental values of two variables x and y.

x	0.5	1.3	2.1	3.5	4.3	5.5
y	3.3	2.5	2	1.5	1.3	1.1

It is known that x and y are related by the equation $y=\frac{a}{x+b}$, where a and b are constants.

(a) On a grid, plot xy against y and obtain a straight line graph. [2]

(b) Use your graph to estimate the value of a and of b. [4]

(c) Obtain the value of the gradient of the straight line obtained when $\frac{1}{y}$ is plotted against x. [2]

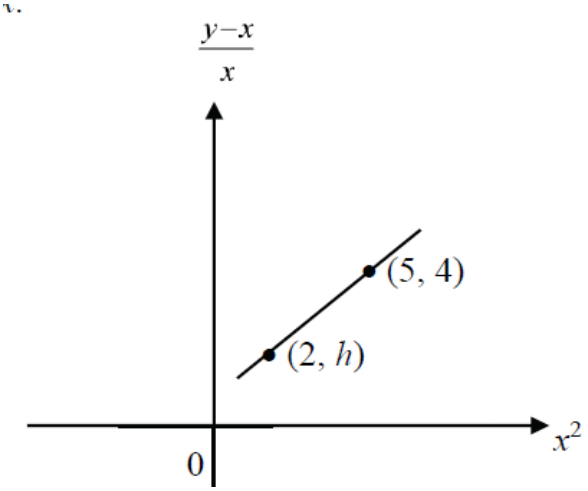
5. (a) The population P, in millions of a city, recorded in the month of January for various years is modelled by the equation $P = 10 + at^n$, where t is the time measured in years from January 2002 and a and n are constants.
The values are tabulated below.

Year	2005	2012	2017	2022
P	20.4	73.2	126.2	188.9

(i) On a grid, plot $lg(P-10)$ against $lg\, t$ for the given data and draw a straight line graph to estimate the values of a and n, giving your answers to one decimal place. [6]

(ii) Use your graph to determine the year in which the population reached 100 million. [2]

(b) The diagram shows part of a straight-line graph passing through the points $(2, h)$ and $(5,4)$, and representing the equations $2x^3 + kx = 3y$, where k and h are constants. Find the value of k and of h. [4]



Answer Key

1(a).	$2^n - 2^{n-3}nx + 2^{n-7}n(n-1)x^2 + \cdots$
1(b).	$n = 4$
1(c).	$a = 32, b = -5$
2(a).	$p = 2, k = 280$
2(b).	-280
4(b).	$a = 8.0 - 8.4, b = 1.8 - 2.2$
4(c).	0.122
5(ai).	$n = 1.5, a = 2.0$
5(aii).	2014
5(b).	$h = 2, k = 5$