S4A TOPICAL INTENSIVE REVISION WEEK 6

Topic: DIFFERENTIATION & APPLICATIONS Total marks: 40

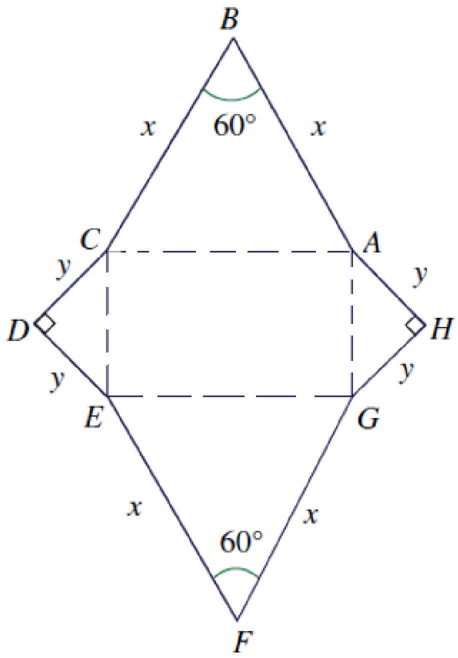
1. The function $f(x)$ is defined by $f(x) = \frac{x}{x^2-3}$. Explain, with working, whether $f(x)$ is an increasing or decreasing function. [3]

2. The equation of a curve is $y = \frac{1}{3}ln(px+3)$, where p is a constant to be determined. The gradient of the tangent to the curve at $x = -\frac{1}{2}$ is parallel to $3y = x$. Find the equation of the normal to the curve at $x = -\frac{1}{2}$. [4]

3. The equation of a curve is $y = Ae^{\frac{1}{2}x} + Be^{-\frac{1}{2}x}$, and that $\frac{dy}{dx} + \frac{3}{2}y = 2e^{\frac{1}{2}x} - 5e^{-\frac{1}{2}x}$. Find the value of each of the constants A and B. [5]

4. A spherical balloon expands at a constant rate of $6cm^3/s$. Find the rate of increase of the surface area when the radius is 3 cm. [6]

5. A piece of wire, 100 cm in length, is bent to form the figure $ABCDEFGH$ as shown.



Given that angle ABC = angle EFG = 60°, angle CDE = angle GHA = 90°, AB = BC = EF = FG = x cm and CD = DE = GH = HA = y cm.

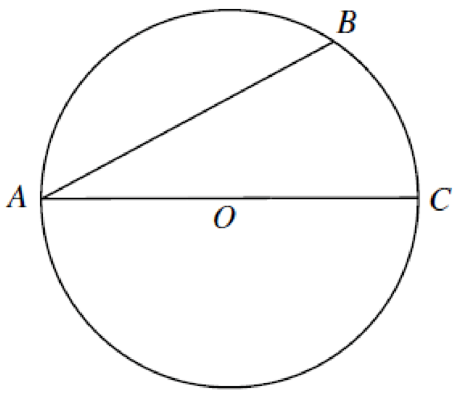
(a) Show that the area of the figure, Pcm^2, is given by

$P = \left(\frac{\sqrt{3}}{2} + 1 - \sqrt{2}\right)x^2 + (25\sqrt{2} - 50)x + 625$ [4]

(b) Find the value of x for which P has a stationary value. [2]

6. Liquid is poured, at a constant rate of $25\pi cm^3/s$, into a hemispherical bowl of radius r cm. When the depth of the liquid directly below the centre of the bowl is x cm, the volume, Vcm^3, of the liquid in the bowl is given by $V = \frac{1}{3}\pi x^2(3r - x)$. It is given that the radius of hemispherical bowl of radius is 12 cm, find
   (a) the time taken for the depth of the liquid directly below the centre of the bowl to reach 6 cm, [3]
   (b) the rate of change of the depth of liquid directly below the centre of the bowl at this time. [4]

7. The figure shows a circular lake, centre O, of radius 2 km. A duck swims across the lake from A to B at 3 km/h and walks clockwise around the edge of the lake from B to C at 4 km/h. If angle BAC = θ radians and the total time taken is T hours,



(i) Show that $T = \frac{1}{3}(4\cos\theta + 3\theta)$, [4]

(ii) Find the maximum value of T. [5]

Answer Key

1.	decreasing
2.	$y = -3x - \frac{3}{2} + \frac{1}{3} ln\, 2$
3.	$A = 1, B = -5$
4.	4 cm^2/s
5(b)	16.2 cm^2
6(a)	14.4 seconds
6(b)	$\frac{dx}{dt} = \frac{25}{108}$cm/s
7(ii)	1.73h