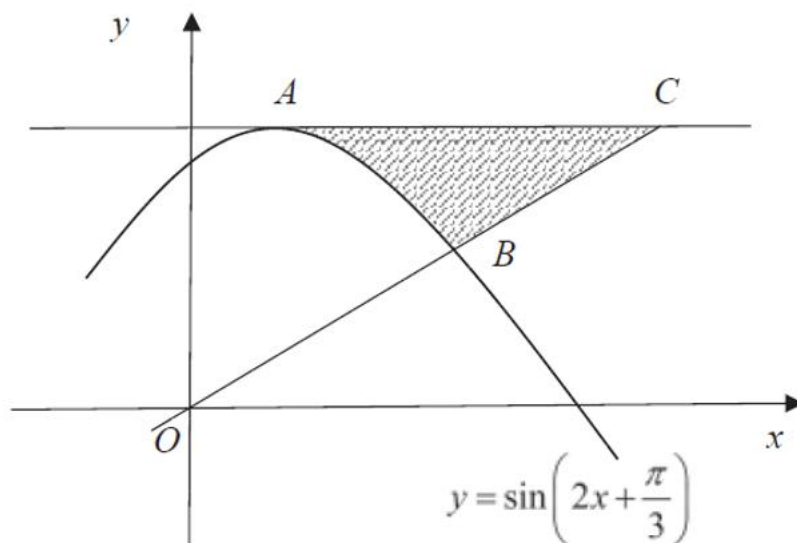


**Topic: INTEGRATION & APPLICATIONS**
**Total marks: 48**

1. For a particular curve,  $\frac{d^2y}{dx^2} = 6x + e^{-\frac{1}{3}x}$ . The curve passes through the point  $P(0,12)$  and the gradient of the curve at P is  $-5$ . Find the equation of the curve. [6]
  
2. a. Given that  $\int_0^3 f(x)dx = a$  and  $\int_0^3 g(x) = -2a$ , where  $a$  is a real number, find the value of  $a$  if  $\int_3^0 f(x)dx + \int_0^3 g(x) = \int_0^3 e^{-3x}dx$  [4]
  
- bi. Given that  $y = \ln\left(\frac{x-3}{x+3}\right)$ , show that  $\frac{dy}{dx} = \frac{6}{x^2-9}$ . [3]
  
- bii. Hence, evaluate  $\int_4^6 \frac{2x+3}{x^2-9}dx$ . [5]
  
3. a. By differentiating  $\ln(\cos x)$ , find  $\int \tan x dx$ . [4]
- b. Given that  $y = x \tan x$ , find  $\frac{dy}{dx}$ . [2]
- c. Using your answers to (a) and (b), find  $\int x \sec^2 x dx$ . [2]
  
4. a. Express  $\frac{1-3x-2x^2}{x(1+2x)^2}$  in partial fractions. [5]
- b. Find the value of  $\int_1^2 \frac{1-3x-2x^2}{x(1+2x)^2} dx$ . [4]

5. The diagram shows part of the curve  $y = \sin\left(2x + \frac{\pi}{3}\right)$  for  $0 \leq x \leq \frac{\pi}{2}$ , where A is the maximum point of the curve. OC is a straight line that cuts the curve at B and AC is parallel to the  $x$ -axis. Given that the  $x$ -coordinate of B is  $\frac{\pi}{4}$ , show that the area of the shaded region is  $\left(\frac{11\pi}{48} - \frac{\sqrt{3}}{4}\right)\text{units}^2$ . [12]



## Answer Key

1.	$y = x + 9e^{-\frac{1}{3}x} - 2x + 3$
2(a).	A=-0.111
2(bii).	1.77
3(a).	$-\ln(\cos x) + c$
3(b).	$x \sec^2 x + \tan x$
3(c).	$x \tan x + \ln(\cos x) + c$
4(a).	$\frac{1}{x} - \frac{3}{1+2x} - \frac{4}{(1+2x)^2}$
4(b).	-0.340